ELL 333

MULTIVARIABLE CONTROL

12.10.2018

— Design.

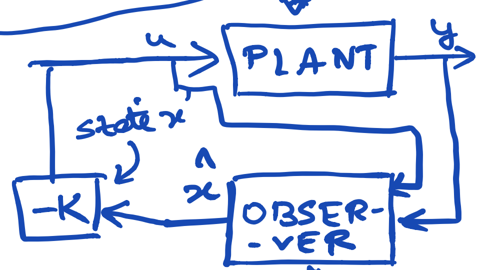
State feedback: $u = -Kx$

→ Reconstruct the state ($\hat{x}$)

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$
$$\hat{y} = C\hat{x}$$

Possible error between y and $\hat{y}$

$$\dot{x} = Ax + Bu$$
$$y = Cx$$



Define $\tilde{x} \triangleq x - \hat{x}$

'~' tilde ; '^' hat

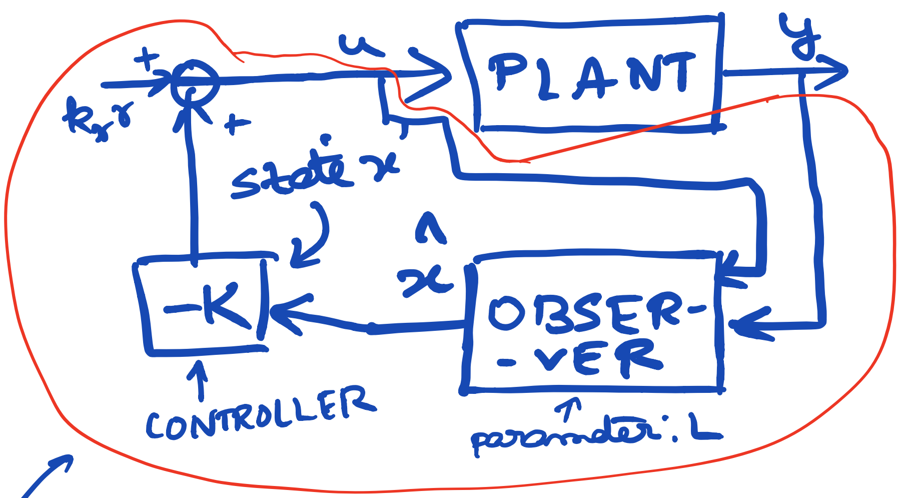
We would like $\tilde{x} \to 0$ so that $x \to \hat{x}$

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}}$$
$$= Ax + \underbrace{Bu - (A\hat{x} + Bu}_{\text{CANCELLED}} + L(\overset{Cx}{y} - \overset{C\hat{x}}{\hat{y}}))$$
$$= A(x - \hat{x}) - L(Cx - C\hat{x})$$
$$= A\tilde{x} - LC\underbrace{(x - \hat{x})}_{\tilde{x}}$$
$$= (A - LC)\tilde{x}$$

$\Rightarrow \dot{\tilde{x}} = (A - LC)\tilde{x}$ whether $\tilde{x} \to 0$, depends on 'A-LC' matrix

Similar questions as controllability/reachability

- How to choose 'L' so that $\hat{x} \to x$?
- Whether it can be done or not?
- Canonical form to help with the analysis
- called 'observability of state'



OVERALL FEEDBACK STRUCTURE

Separation principle:

$$\dot{x} = Ax + Bu \qquad u = -K\hat{x} + k_r r$$

this is $\hat{x}$ because of the reconstruction of state that is required

$$y = Cx$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$
$$\hat{y} = C\hat{x}$$

$\Rightarrow$ $(x, \hat{x})$ co-ordinates

$$\dot{x} = Ax + B(-K\hat{x} + k_r r)$$
$$\dot{\hat{x}} = A\hat{x} + B(-K\hat{x} + k_r r) + LC(x - \hat{x})$$

Using $\tilde{x} = x - \hat{x}$,

$(x, \tilde{x})$ co-ordinates

$$\dot{x} = Ax - BK(x - \tilde{x}) + Bk_r r$$
$$\Rightarrow = (A - BK)x + BK\tilde{x} + Bk_r r$$
$$\dot{\tilde{x}} = (A - LC)\tilde{x}$$

$$\frac{d}{dt}\begin{bmatrix} x \\ \tilde{x} \end{bmatrix}_{2n\times 1} = \left[\begin{array}{c|c} A-BK & BK \\ \hline 0_{n\times n} & A-LC \end{array}\right]_{2n\times 2n} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix}_{2n\times 1} + \begin{bmatrix} Bk_r r \\ 0 \end{bmatrix}$$