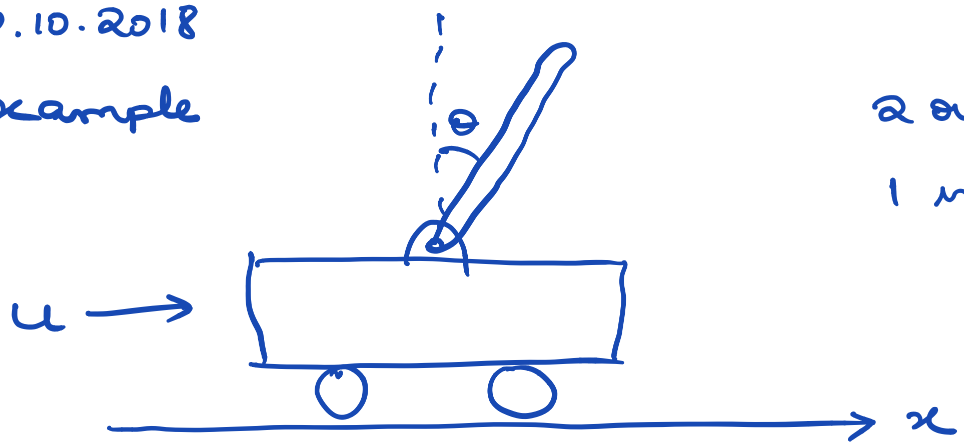
ELL333

MULTIVARIABLE CONTROL

30.10.2018

Example




position angle

2 outputs : $x, \theta$

1 input : $u$

force

Design Objective: Design '$u$' to ensure '$x$' reaches a reference position '$x_r$' and '$\theta$' reaches a reference angle '$\theta_r$' (typically $\theta_r = 0$).

First, can this be done?

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Controllability : A system $\dot{x} = Ax + Bu$, $y = Cx + Du$, ~~and final time '$t_f$'.~~ is controllable if for any initial time $t_0$ and any initial state $x_0 = x(t_0)$, there is an input $u$ and final time '$t_f$' such that $x(t_f) = 0$.

Reachability : A system $\dot{x} = Ax + Bu$, $y = Cx + Du$ is reachable if for any initial time '$t_0$' ~~and final time '$t_f$'~~, any initial state '$x_0$' and final state '$x_f$', there is an input $u$ and final time $t_f$ such that $x(t_0) = x_0$ and $x(t_f) = x_f$.

These can be shown to be equivalent to the condition

$$\text{rank} \left[ B \,\vdots\, AB \,\vdots\, \cdots \, A^{n-1} B \right] = n.$$

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For example above, see Example 7.4

Computation: $\text{rank} \left[ B \;\; AB \;\; A^2B \;\; A^3B \right] = 4$

- System is 'controllable' / 'reachable'
  ('we have not shown this equivalence rigorously')
- We can change eigenvalues to arbitrary locations using the linear control

$$u = -k_1 x - k_2 \theta - k_3 \dot{x} - k_4 \dot{\theta}$$

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Check Fig 7.3 to see an example of uncontrollable / unreachable system.

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All this assumes that the states are known.

Recall the overall control structure from lecture on separation principle, (12.10.2018)

$$\frac{d}{dt}\underbrace{\begin{bmatrix} x \\ \tilde{x} \end{bmatrix}}_{2n\times 1} = \underbrace{\left[\begin{array}{c:c} A-BK & BK \\ \hdashline 0_{n\times n} & A-LC \end{array}\right]}_{2n\times 2n} \underbrace{\begin{bmatrix} x \\ \tilde{x} \end{bmatrix}}_{2n\times 1} + \begin{bmatrix} Bk_r r \\ 0 \end{bmatrix}$$

So far, we focussed on 'How to choose $K$ to arbitrarily place eigenvalues of $A-BK$, and whether this can be done or not?'

Now, similar question on choosing $L$?

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Consider $(A-LC)^T = A^T - C^T L^T$