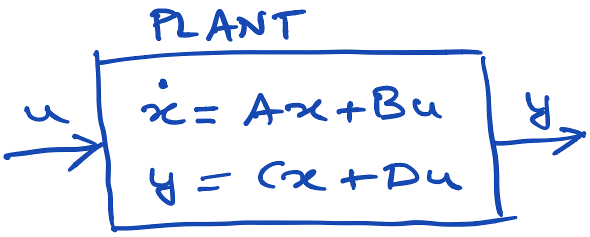
ELL333

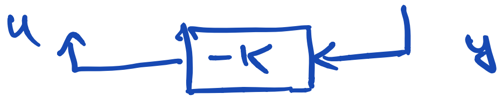
MULTIVARIABLE CONTROL

31.10.2018

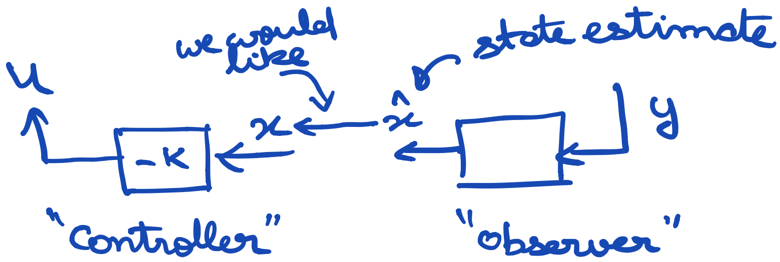


Controllers

Possibility 1 :



Possibility 2 :

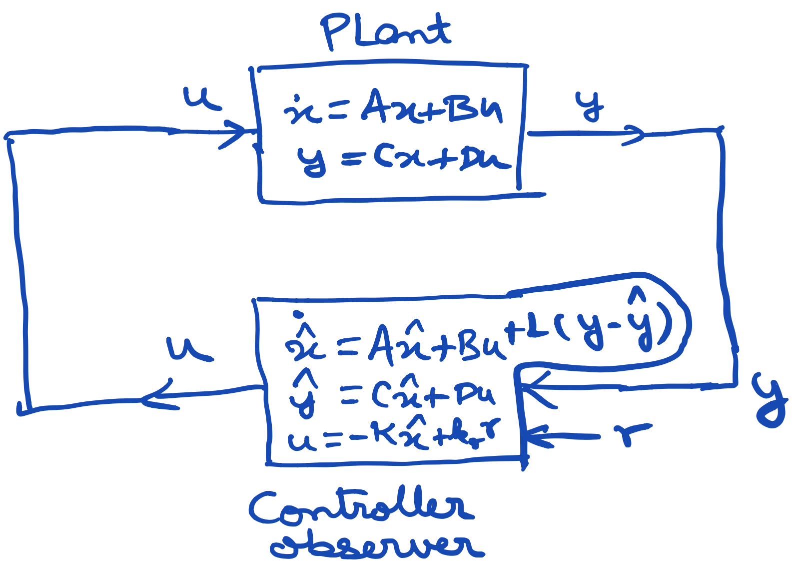


eqn : $u = -Kx + k_r r$

reference

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$
$$\hat{y} = C\hat{x} + Du$$

Combined Block Diagram of Plant + Observer Controller



$$\frac{d}{dt}\begin{bmatrix} x \\ \tilde{x} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix}\begin{bmatrix} x \\ \tilde{x} \end{bmatrix} + \begin{bmatrix} Bk_r \\ 0 \end{bmatrix} r$$

$\tilde{x} = x - \hat{x}$

Both K and L have to be designed to place eigenvalues of $A-BK$ and $A-LC$, respectively, to suitable desired locations

And whether this can be done?

K can be found to do above if $\text{rank}[B \; AB \ldots A^{n-1}B] = n$.
(shown for single input)

What about the same question for L:
Can L be chosen to arbitrarily place eigenvalues of $A-LC$ to desired locations? and How?

What could these be?

$$\dot{x} = (A-BK)x + BK\tilde{x} + Bk_r r$$

Because of this term, we may want $\tilde{x} \to 0$ at a desired speed.

$$\dot{\tilde{x}} = \underbrace{(A-LC)}_{} \tilde{x}$$

eigenvalues in LHP $\Rightarrow \tilde{x} \to 0$

To solve eigenvalue assignment of $A-LC$ using L, we note the following,

$$(A-LC)^T = A^T - (LC)^T = \underset{\downarrow \atop \hat{A}}{A^T} - \underset{\downarrow \atop \hat{B}}{C^T} \, \underset{\downarrow \atop \hat{K}}{L^T}$$

[We are doing this because eigenvalues of a matrix and its transpose are the same.
So assigning eigenvalues of $A-LC$ is same as assigning eigenvalues of $(A-LC)^T$.]

10 states, 2 inputs, 3 outputs

A : 10×10　　K : 2×10

B : 10×2　　L : 10×3

C : 3×10

(D=0)

Three things we did for eigenvalue assignment of $\hat{A}-\hat{B}\hat{K}$ using $\hat{K}$ (for single input)

1. $\hat{K} = [\ \ ][\ \ ]^{-1}[\ \ ]$ exists iff this matrix is invertible.

This matrix $= \begin{bmatrix} \hat{B} & \hat{A}\hat{B} & \dots & (\hat{A})^{n-1}\hat{B} \end{bmatrix}$

in terms of A and C

$$= \begin{bmatrix} C^T & A^T C^T & \dots & (A^T)^{n-1} C^T \end{bmatrix}$$

Rank of this needs to be 'n'.

2. Canonical form for $\hat{A}$, $\hat{B}$

$$\hat{A} = \begin{bmatrix} -a_1 & -a_2 & \dots & -a_{n-1} \\ 1 & 0 & \dots & 0 \\ 0 & 0 & \dots 1 & 0 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\Rightarrow A^T = \begin{bmatrix} -a_1 & -a_2 & \dots & -a_{n-1} \\ & \dots & & \\ & & & \end{bmatrix}, \quad C^T = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} -a_1 & 1 & \dots & 0 \\ -a_2 & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ -a_{n-1} & 0 & & 1 \\ & & & 0 \end{bmatrix}, \quad C = [1\ 0 \dots 0]$$

3. Definition of controllability motivated by the following potential basis of state space $\{\hat{B}, \hat{A}\hat{B}, \ldots (\hat{A})^{n-1}\hat{B}\}$

$$\equiv \{C^T, A^T C^T, \ldots (A^T)^{n-1} C^T\}$$

↳ Observability