ELL333 / MULTIVARIABLE CONTROL

14.11.2018

$x = [p \; r \; \beta \; \phi]^T = \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix}$, $y = Cx$

a) only p is measured $\Rightarrow C = [1 \; 0 \; 0 \; 0]$

b) only β is measured $\Rightarrow C = [0 \; 0 \; 1 \; 0]$

c) both p & β measured $\Rightarrow C = ?$

$C = [1 \; 0 \; 1 \; 0] \Rightarrow y = Cx = p + \beta$

$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow y = \begin{bmatrix} p \\ \beta \end{bmatrix}$

(PBH Test)

$\text{rank}[B \; AB \ldots A^{n-1}B] = n \iff$

$\text{rank}[\lambda I - A \quad B] = n \quad \forall \lambda \in \mathbb{C}$

($\Rightarrow$) Suppose $\text{rank}[\lambda I - A \quad B] < n$

$\Rightarrow \exists \; x \; (\neq 0)$ such that $x^T[\lambda I - A \quad B] = 0$

$\Rightarrow x^T(\lambda I - A) = 0$ & $x^T B = 0$

$\Rightarrow \lambda x^T = x^T A$ & $x^T B = 0$

$x^T[B \; AB \ldots A^{n-1}B] = [x^T B \;\; x^T AB \; \ldots \; x^T A^{n-1} B]$

$$= [\,0 \quad \lambda x^T B \quad \lambda^2 x^T B \quad \cdots \quad \lambda^{n-1} x^T B\,]$$

$$= 0$$

$\Rightarrow$ rank $[B \quad AB \quad \cdots A^{n-1}B] < n$, a contradiction