ELL333 **QUIZ 1** MARKS = 7 DURATION = 20 MINUTES

Q. The equations of motion of a "Lorenz System" are given below. This is a model of weather and may show chaos. What are the equilibrium points? Linearize the model about each of the equilibrium points.

$$\dot{x} = \sigma(y - x) \qquad x, y, z \rightarrow \text{states}$$

$$\dot{y} = x(\rho - z) - y \qquad \sigma, \rho, \beta \rightarrow \text{parameters}$$

$$\dot{z} = xy - \beta z \qquad \underline{\text{NO}} \text{ input.} \qquad \sigma > 0, \rho > 1, \beta > 0$$

NAME =

ENTRY No. =

Answer.

Equilibrium points $\Rightarrow \dot{x} = \dot{y} = \dot{z} = 0$ 1

$\dot{x} = 0 \Rightarrow x = y$

$\dot{y} = 0 \Rightarrow x(\rho - z) - y = 0 \Rightarrow x\{\rho - z - 1\} = 0$

$\Rightarrow x = 0 = y$ (Case 1) or $z = \rho - 1$ (Case 2) 1

$\dot{z} = 0 \Rightarrow xy - \beta z = 0$

$\Rightarrow xy = \beta z$.

Case 1: $\Rightarrow z = 0$

Case 2: $\Rightarrow x^2 = \beta z$

$\Rightarrow x^2 = \beta(\rho - 1)$

$\Rightarrow x = y = \pm\sqrt{\beta(\rho - 1)}$

$\therefore$ Equilibrium points are

$(0, 0, 0)$, $(+\sqrt{\beta(\rho-1)}, +\sqrt{\beta(\rho-1)}, \rho - 1)$, $(-\sqrt{\beta(\rho-1)}, -\sqrt{\beta(\rho-1)}, \rho - 1)$

#1 1 #2 1 #3

At an equilibrium point (x_0, y_0, z_0), linearization is

$$\begin{bmatrix} \Delta\dot{x} \\ \Delta\dot{y} \\ \Delta\dot{z} \end{bmatrix} = \begin{bmatrix} -\sigma & \sigma & 0 \\ \rho - z_0 & -1 & -x_0 \\ y_0 & x_0 & -\beta \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$$

1.5

$$\begin{bmatrix} -\sigma & \sigma & 0 \\ \rho & -1 & 0 \\ 0 & 0 & -\beta \end{bmatrix}$$

at #1 0.5

$$\begin{bmatrix} -\sigma & \sigma & 0 \\ 1 & -1 & -\sqrt{\beta(\rho-1)} \\ \sqrt{\beta(\rho-1)} & \sqrt{\beta(\rho-1)} & -\beta \end{bmatrix}$$

at #2 0.5

$$\begin{bmatrix} -\sigma & \sigma & 0 \\ 1 & -1 & \sqrt{\beta(\rho-1)} \\ -\sqrt{\beta(\rho-1)} & -\sqrt{\beta(\rho-1)} & -\beta \end{bmatrix}$$

at #3 0.5