ELL333 QUIZ 3 MARKS=7 DURATION = 20 min

Q. Fill in the blanks ( [ ] )

NAME = , ENTRY NUMBER =

Let us investigate the stability, using Lyapunov's method, of the fixed point $(x, y) = (0, 0)$ of the system,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad x, y \in \mathbb{R}$$

Recall that, to show stability, we need to find a $V(x,y)$ (Lyapunov function) that satisfies,

① $V(x,y) > 0 \quad \forall (x,y) \in \mathbb{R}^2 - \{(0,0)\}$ and $V(0,0) = 0$

② $\dot{V}(x,y) \leq 0 \quad \forall (x,y) \in \mathbb{R}^2$

Consider a candidate Lyapunov function

$V(x,y) = ax^2 + by^2$, where, to satisfy ①, a and b have to be [ ] ← What is condition on a and b?

$\Rightarrow \dot{V}(x,y) =$

Suppose, we set $\dot{V}(x,y) \equiv -\alpha x^2 - \beta y^2$, where, to satisfy ② $\alpha$ and $\beta$ have to be [ ].

↑ What is condition on $\alpha$ and $\beta$?

One possible set of values of a and b are

[ a = , b = ]

This shows that $(0,0)$ is stable.

ELL333 QUIZ 3 MARKS=7 DURATION = 20 min

Q. Fill in the blanks ( [ ] )

NAME = , ENTRY NUMBER =

Let us investigate the stability, using Lyapunov's method, of the fixed point $(x,y)=(0,0)$ of the system,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad x, y \in \mathbb{R}$$

Recall that, to show stability, we need to find a $V(x,y)$ (Lyapunov function) that satisfies,

① $V(x,y) > 0 \quad \forall (x,y) \in \mathbb{R}^2 - \{(0,0)\}$ and $V(0,0) = 0$

② $\dot{V}(x,y) \leq 0 \quad \forall (x,y) \in \mathbb{R}^2$

Consider a candidate Lyapunov function

$V(x,y) = ax^2 + by^2$, where, to satisfy ①, a and b have to be (1) strictly positive ← What is condition on a and b?

$$\Rightarrow \dot{V}(x,y) = 2ax\dot{x} + 2by\dot{y}$$

(3)

$$= 2ax(-3x+2y) + 2by(-x-y)$$

$$= x^2(-6a) + xy(4a-2b) + y^2(-2b)$$

Suppose, we set $\dot{V}(x,y) \equiv -\alpha x^2 - \beta y^2$, where, to satisfy ② $\alpha$ and $\beta$ have to be (1) positive.

↑ What is condition on $\alpha$ and $\beta$?

One possible set of values of a and b are

(2) $a = 1, \; b = 2$. $\left\{ \because 4a - 2b = 0 \Rightarrow b = 2a \right.$

$\left. \alpha = 6a > 0, \; \beta = 2b > 0 \right.$

This shows that $(0,0)$ is stable.