ELL 333 QUIZ 4 MARKS = 7 DURATION = 20 min

NAME = , ENTRY NUMBER =

Q. Consider the system $\dot{x} = Ax + Bu$, where

$$A = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \quad \begin{matrix} \lambda_1 \neq \lambda_2 \\ \lambda_1 \neq \lambda_3 \\ \lambda_2 \neq \lambda_3 \end{matrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

Suppose $u = -Kx$. (state feedback)

a) What is the condition on b_1, b_2, b_3 for the eigenvalues of the system with state feedback to be placed at arbitrarily locations through suitable choice of K?

b) Based on a), give a generalized statement for when A is a $n \times n$ diagonal matrix with distinct eigenvalues and B is an $n \times 1$ column vector with j^{th} element b_j, $j = 1, 2, \ldots n$

A. The condition for the eigenvalues to be placed at arbitrary locations is

$$\text{rank}\left(\begin{bmatrix} B & AB & A^2B \end{bmatrix}\right) = n$$

$$\Rightarrow \text{rank}\left(\underbrace{\begin{bmatrix} b_1 & \lambda_1 b_1 & \lambda_1^2 b_1 \\ b_2 & \lambda_2 b_2 & \lambda_2^2 b_2 \\ b_3 & \lambda_3 b_3 & \lambda_3^2 b_3 \end{bmatrix}}_{= W_c}\right) = 3$$

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$$\Rightarrow \det(W_c) \neq 0$$

$$\det(W_c) = \begin{vmatrix} b_1 & \lambda_1 b_1 & \lambda_1^2 b_1 \\ b_2 & \lambda_2 b_2 & \lambda_2^2 b_2 \\ b_3 & \lambda_3 b_3 & \lambda_3^2 b_3 \end{vmatrix} = \begin{vmatrix} b_1 & \lambda_1 b_1 & \lambda_1^2 b_1 \\ 0 & b_2(\lambda_2 - \lambda_1) & b_2(\lambda_2^2 - \lambda_1^2) \\ 0 & b_3(\lambda_3 - \lambda_1) & b_3(\lambda_3^2 - \lambda_1^2) \end{vmatrix}$$

$$= b_1 b_2 b_3 \left[(\lambda_2 - \lambda_1)(\lambda_3^2 - \lambda_1^2) - (\lambda_3 - \lambda_1)(\lambda_2^2 - \lambda_1^2)\right]$$

$$= b_1 b_2 b_3 \left[(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)(\lambda_3 + \lambda_1) - (\lambda_3 - \lambda_1)(\lambda_2 - \lambda_1)(\lambda_2 + \lambda_1)\right]$$

$$= b_1 b_2 b_3 (\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)$$

This is non-zero if none of b_1, b_2, b_3 are zero. 3

For a $n \times n$ diagonal A matrix with distinct eigenvalues, the condition for controllability is that none of $b_1, b_2, \ldots b_n$ are zero

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