ELL301

23.01.2019

Voltmeter

"Linear version" of MOVING IRON

$$m\ddot{x} + b\dot{x} + kx = K V^2$$

mass of moving part

damping: mechanical, magnetic (eddy current)

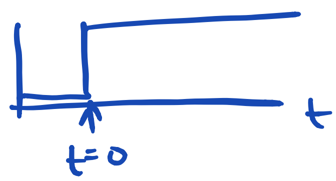
restoring spring

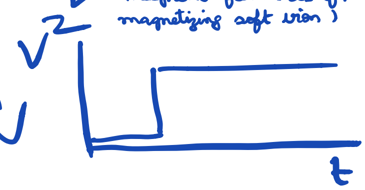
geometric, electric factors

voltage to be measured

square comes from the "double" effect of magnetic field (itself + magnetizing soft iron)

dc voltage, $V \rightarrow$





One way $\rightarrow$ Laplace transform

$(x(0) = 0 = \dot{x}(0)$

$$m s^2 X(s) + b s X(s) + k X(s) = K V^2 \cdot \frac{1}{s}$$

$$\Rightarrow X(s) = \frac{KV^2}{s} \cdot \frac{1}{ms^2 + bs + k}$$

- Final value / steady-state value ?

Use final value theorem

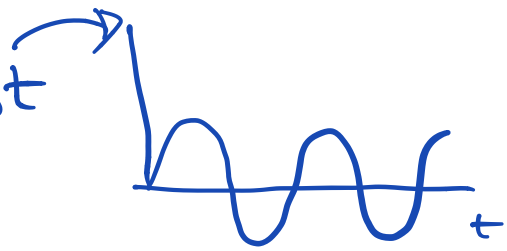
$$x_{ss} = \lim_{s \to 0} s\, X(s) = \lim_{s \to 0} s \cdot \frac{KV^2}{s} \cdot \frac{1}{ms^2 + bs + k}$$

$$= \frac{K}{k} \cdot V^2$$

- Dynamics can be obtained from standard second order system dynamics
  $\longrightarrow$ depend on K, m, b, k

- Note dependence of x_{ss} on V^2
  $\rightarrow$ a nonlinear scale may be needed

ac voltage, $V = V_0 \sin\omega t$



What is response of x?

$$m\ddot{x} + b\dot{x} + kx = K V_0^2 \sin^2\omega t$$
$$= K\frac{V_0^2}{2}(1 - \cos 2\omega t)$$

Response can be split into

$\frac{KV_0^2}{2} \longrightarrow$ steady state of $\underline{\frac{K}{k} \cdot \frac{V_0^2}{2}}$ + dynamics

dc part

$\frac{KV_0^2}{2}\cos 2\omega t \longrightarrow$ (amplitude) $\cos(2\omega t + \text{phase})$

$\Big\downarrow \mathcal{L}$

$\frac{KV_0^2}{2} \cdot \frac{s}{s^2 + (2\omega)^2}$

$\hookrightarrow \underline{\frac{KV_0^2}{2} \cdot \left| \frac{1}{ms^2 + bs + k} \right| \Big|_{at\ s = j2\omega}}$

ac part amplitude

For needle to come close to rest, want $\left| \frac{1}{ms^2 + bs + k} \right|_{at\ s = j2\omega} \ll 1$

Note: For a linear system (stable)

$$A\cos\omega_0 t \longrightarrow \boxed{H(s)} \longrightarrow A\,|H(j\omega_0)|\cos(\omega_0 t + \angle H(j\omega_0))$$

Final value theorem

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s), \quad \text{if } (\text{limit exists})$$

$$\left[\ f(t) \xrightarrow{\mathcal{L}} F(s)\ \right]$$

$\equiv$ $F(s)$ should have poles in LHP, or atmost one on j-axis

if $f(t) = \cos\omega t$, final value? not defined

$$\lim_{s\to 0} s\cdot\frac{s}{s^2+\omega^2} = 0$$

if $f(t) = e^{at}$, final value? exists if $a<0$

$$\lim_{s\to 0} s\cdot\frac{1}{s+a} = 0$$

if $f(t) =$ unit step, final value?

$$\lim_{s\to 0} s\cdot\frac{1}{s} = 1$$

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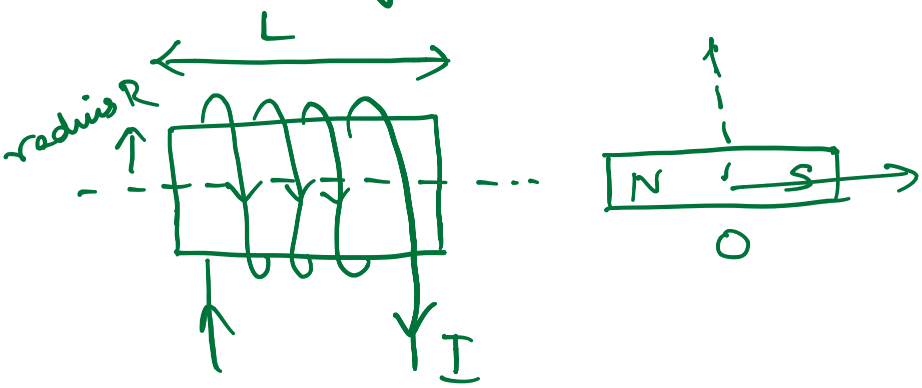

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Biot-Savart Law

$$\frac{\mu_0 n I}{2} (\cos\theta_2 - \cos\theta_1)$$

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convert angle to distance

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$$\longrightarrow \frac{\mu_0 n I}{2} \left[f_1(z) - f_2(z) \right]$$

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— non-uniformity drives motion

$$U = - \vec{\mu_n} \cdot \vec{B}$$

Force, $F = - \partial_z U$

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not much time to calculate

— geometric factors

? For more information on this, please contact Soumyadip Banerjee