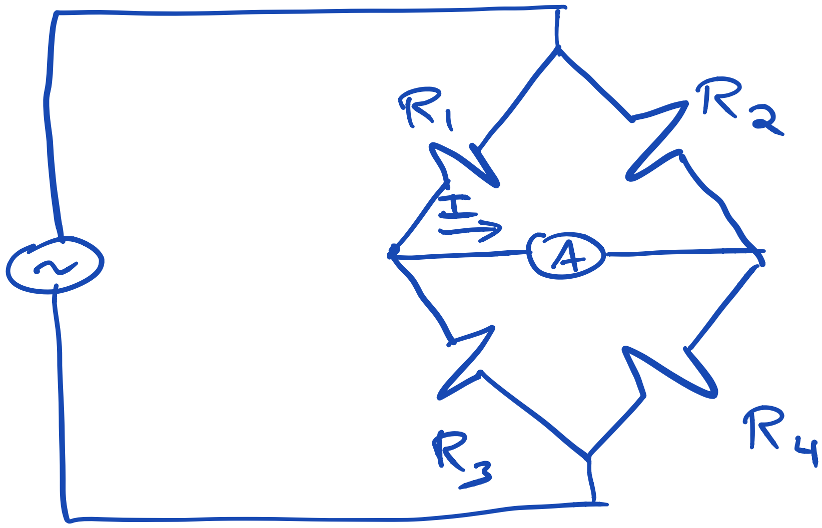
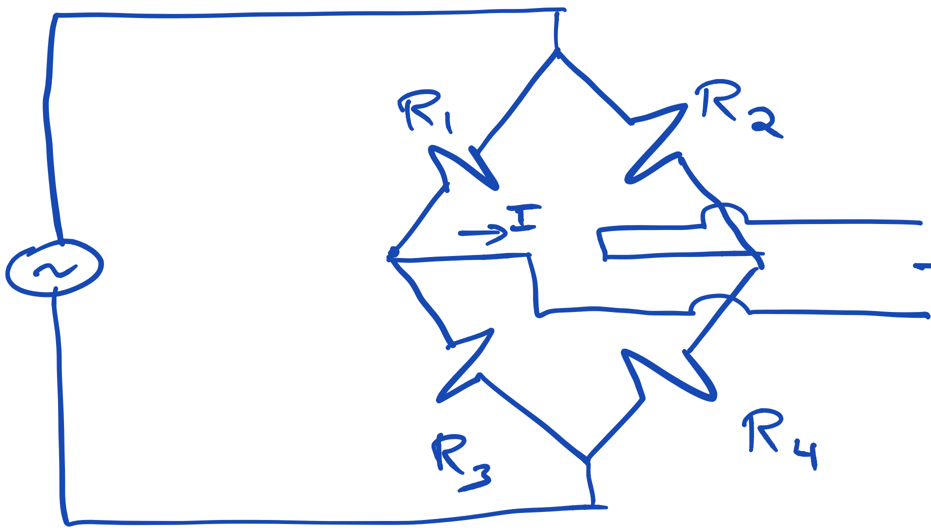
ELL301

27.02.2019

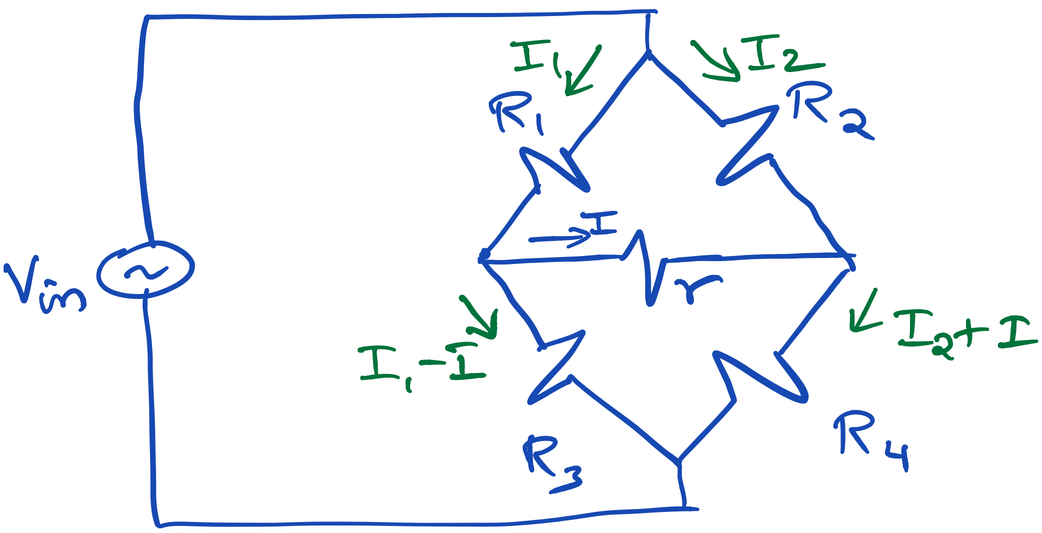
Wheatstone Bridge



in sensor uses



To amplifier



r is the input resistance of the amplifier part

Suppose, one of the arms, say R_1 is a sensor and due to sensing the physical variable, its value changes $R_1 \to R_1 + \Delta R$, then how much does the current I change?

$$-V_{in} + I_1 R_1 + (I_1 - I) R_3 = 0$$

$$\Rightarrow I_1 = \frac{V_{in}}{R_1 + R_3} + I \cdot \frac{R_3}{R_1 + R_3} \quad \text{--- (1)}$$

$$\text{Ily} \quad -V_{in} + I_2 R_2 + (I_2 + I) R_4 = 0$$

$$\Rightarrow I_2 = \frac{V_{in}}{R_2 + R_4} - I \cdot \frac{R_4}{R_2 + R_4} \quad \text{--- (2)}$$

$$\text{Ily} \quad -V_{in} + I_1 R_1 + I r + (I_2 + I) R_4 = 0$$

Replace I_1 from (1) & I_2 from (2),

$$\Rightarrow -V_{in} + V_{in} \cdot \frac{R_1}{R_1 + R_3} + I \frac{R_1 R_3}{R_1 + R_3}$$

$$+ I (r + R_4)$$

$$+ V_{in} \frac{R_4}{R_2 + R_4} - I \frac{R_4^2}{R_2 + R_4} = 0$$

$$\Rightarrow \quad V_{in}\left[\underbrace{-1+\frac{R_1}{R_1+R_3}}_{\frac{-R_3}{R_1+R_3}}+\frac{R_4}{R_2+R_4}\right]$$

$$+\, I\left[\frac{R_1 R_3}{R_1+R_3}+r+\underbrace{R_4-\frac{R_4^2}{R_2+R_4}}_{\frac{R_4 \cdot R_2}{R_2+R_4}}\right]=0$$

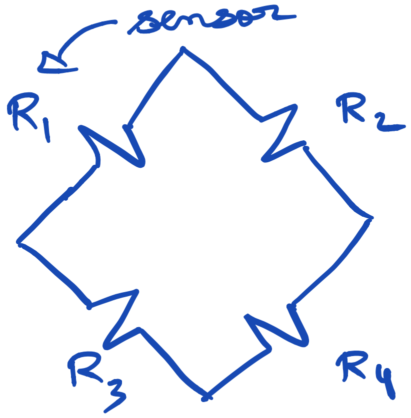
$$\Rightarrow \quad V_{in}\left[\frac{-R_3}{R_1+R_3}+\frac{R_4}{R_2+R_4}\right]$$

$$+\, I\left[\frac{R_1 R_3}{R_1+R_3}+r+\frac{R_2 R_4}{R_2+R_4}\right]=0$$

$$\Rightarrow \quad I = \frac{V_{in}\left[\frac{R_3}{R_1+R_3}-\frac{R_4}{R_2+R_4}\right]}{\left[r+\frac{R_1 R_3}{R_1+R_3}+\frac{R_2 R_4}{R_2+R_4}\right]}$$

$$\Rightarrow \quad I = V_{in}\,\frac{R_3(R_2+R_4)-R_4(R_1+R_3)}{r(R_1+R_3)(R_2+R_4)+R_1R_3(R_2+R_4)+R_2R_4(R_1+R_3)}$$

$$\Rightarrow I = V_{in}\,\frac{R_2R_3-R_1R_4}{r(R_1+R_3)(R_2+R_4)+R_1R_3(R_2+R_4)+R_2R_4(R_1+R_3)}$$



Initially, $R_1 = R_2 = R_3 = R_4 = R$.

$\Rightarrow I = 0$. as $R_2 R_3 = R_1 R_4$

Due to sensing,

$R_1 = R \longrightarrow R_1 = R + \Delta R$

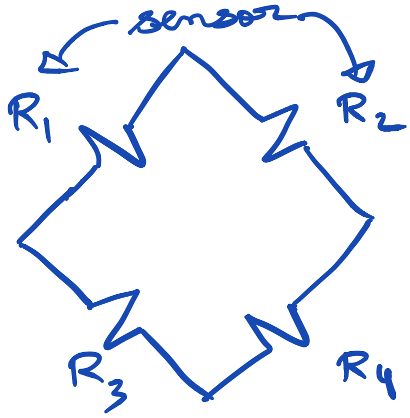
$\Delta R << R$

then, what is the change in current, ΔI?

$$\Delta I = V_{in} \frac{R_2 R_3 - (R+\Delta R) R_4}{r(R+\Delta R+R_3)(R_2+R_4) + (R+\Delta R)R_3(R_2+R_4) + R_2 R_4 (R+\Delta R+R_3)}$$

$$= V_{in} \frac{-R_4 \Delta R}{r(R+\Delta R+R_3)(R_2+R_4) + (R+\Delta R)R_3(R_2+R_4) + R_2 R_4 (R+\Delta R+R_3)}$$

$$\approx V_{in} \frac{-R_4 \Delta R}{r(R+R_3)(R_2+R_4) + (R+\quad)R_3(R_2+R_4) + R_2 R_4 (R+\quad R_3)}$$



$R_1 : R \to R + \Delta R$

$R_2 : R \to R + \Delta R$

ΔI ?

$= 0$

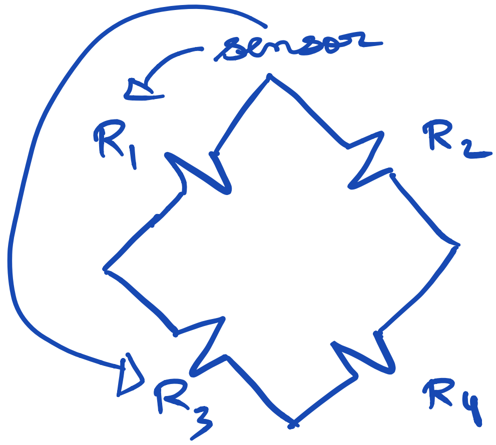
[∵

$R_2 R_3 - R_1 R_4$

$= (R+\Delta R) R_3$

$\quad - (R+\Delta R) R_4$

$= 0$]



$R_1 : R \to R + \Delta R$

$R_3 : R \to R + \Delta R$

ΔI ?

$= 0$

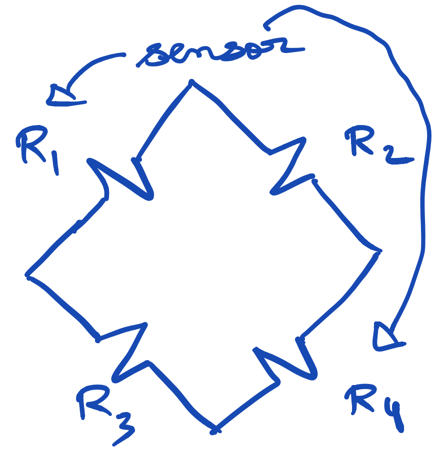
[∵

$R_2 R_3 - R_1 R_4$

$= (R+\Delta R) R_2$

$\quad - (R+\Delta R) R_4$

$= 0$]



$R_1 : R \to R + \Delta R$

$R_4 : R \to R + \Delta R$

ΔI ?

= add up?

[∵

$R_2 R_3 - R_1 R_4$

$= R_2 R_3 - (R+\Delta R)^2$

$= R_2 R_3 - R^2$

$\quad - 2\Delta R . R - (\Delta R)^2$

$\approx -2 \Delta R . R$

Advantage:

Makes the bridge intrinsically robust to disturbances which may cause $\Delta R \neq 0$

Advantage:

Increases the gain of the bridge