ELL301
09.04.2019 (contd)

$$Q = \{Q\}[Q]$$

$\{Q\}$ → Number

$[Q]$ → Unit
↳ Standards

→ Numbers, in the context of instruments and measurement, have a lot of information in them,
↳ numerical value
↳ extent to which numerical value can be stated → accuracy, precision, resolution of instrument
↳ have an uncertainty associated with them

different errors can be modelled as noise

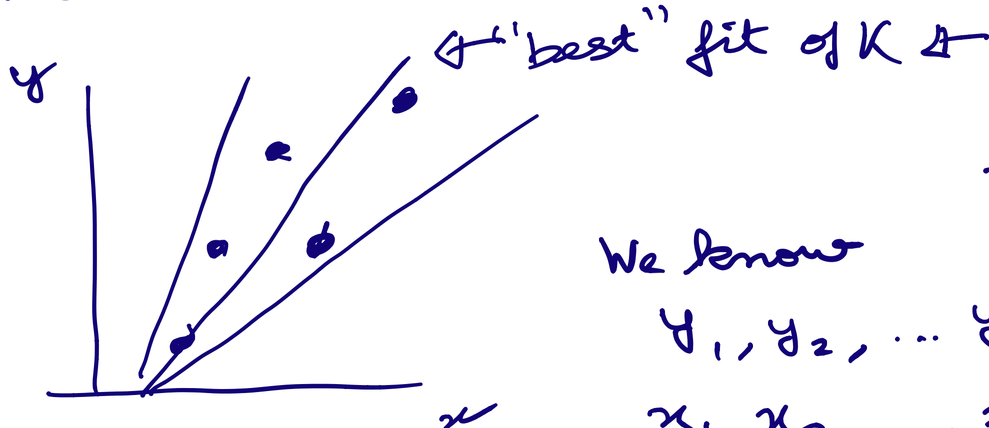
Example: Least Squares

model → $y = K x + n$ → noise

y → measurement

K → slope which we want

x → measured/input



we want to find the K that is 'best' fit to the data

We know

$y_1, y_2, \ldots y_N$

$x_1, x_2, \ldots x_N$

$$\text{Cost}(K) = \sum_{i=1}^{N} (y_i - K x_i)^2$$

differentiate w.r.t K, to find minimum,

$$\frac{\partial \, \text{Cost}}{\partial K} = 0$$

$$\Rightarrow \sum_{i=1}^{N} 2(y_i - K x_i)(-x_i) = 0$$

$$\Rightarrow \sum_{i=1}^{N} x_i y_i - K \sum_{i=1}^{N} x_i^2 = 0$$

$$\Rightarrow K = \frac{\sum_{i=1}^{N} x_i y_i}{\sum_{i=1}^{N} x_i^2}$$

call this $\hat{K}$

- Is this minimum cost?
  check $\frac{\partial^2 \text{cost}}{\partial K^2} > 0$
- When does such a K exist?
  $\sum_{i=1}^{N} x_i^2 = 0$

We have

$$\hat{K} = \frac{\sum_{i=1}^{N} x_i y_i}{\sum_{i=1}^{N} x_i^2}$$

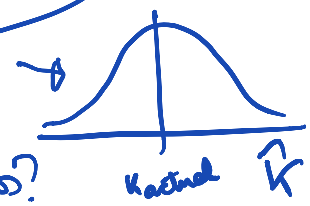
and measurement model is $y = Kx + n$

Exp I → $\{x_i, y_i\}_{i=1}^{100} \to \hat{K}$

Different experiments

$\{x_i, y_i\}_{i=1}^{100} \to \hat{K}$ ... $\{x_i, y_i\}_{i=1}^{100} \to \hat{K}$

Exp 2   Exp 100



desired distribution?

$E(\hat{K}) = ?$ unbiased, low variance

$E(\hat{K}) = K_{actual}$

variance as small as possible

estimator terminology

Is $\hat{K}$ an unbiased estimator of K?

$E(\hat{K}) = ?$

variance of estimator $= ?$

Unbiased (relative to the model)

$$E\{(\hat{K} - K)\} = 0$$