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ELL301

The Least Squares method is best method for measurement where noise is (independent) Gaussian in the sense that the corresponding estimator achieves minimum possible variance.

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error $\neq$ mistake

in context of measurement errors are inherent.

↳ Random error

- measurement noise

N readings of a quantity of interest

$y_1, y_2, \dots y_N$

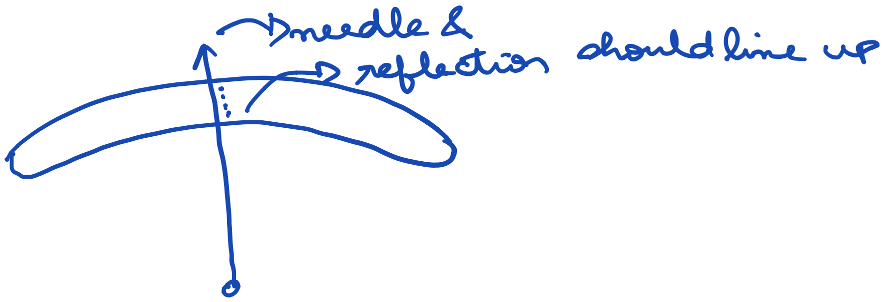
Mean, $\mu_y = \frac{1}{N} \sum_{i=1}^{N} y_i$

Standard Deviation $\sigma_y = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \mu_y)^2}$

The summary of measurement is represented as $\mu_y \pm \sigma_y$ (N readings)

↳ Gross Error / Human Error

↳ Parallax Error



↳ Environmental errors

Temperature effects instrument properties

⇒ this contributes to different readings at different times

↳ Systematic error

→ instrument characteristics drift over time

→ loading effects

Propagation of uncertainty.

$$y = f(x_1, x_2, \ldots x_N)$$

each x_i has a mean value μ_i and a standard deviation σ_i.

What is the mean of y and its standard deviation?

Suppose,

$$y = \frac{1}{N}(x_1 + x_2 + \cdots x_N)$$

$$x_i = \mu_i \pm \sigma_i \Rightarrow y = \mu_y \pm \sigma_y \,?$$

μ_y, mean of $y = \frac{1}{N}(\mu_1 + \mu_2 + \dots \mu_N)$

σ_y, standard deviation of y?

$$\sigma_y^2 = \frac{1}{N}(y - \mu_y)^2$$

$$= \frac{1}{N}\left(\frac{1}{N}(x_1 + x_2 + \dots + x_N) - \frac{1}{N}(\mu_1 + \mu_2 + \dots + \mu_N)\right)^2$$

$$= \frac{1}{N}\left[\frac{1}{N}(x_1 - \mu_1) + \frac{1}{N}(x_2 - \mu_2) + \dots \frac{1}{N}(x_N - \mu_N)\right]^2$$

$$= \frac{1}{N}\Big[\big(\underbrace{\tfrac{1}{N}(x_1 - \mu_1)\big)^2}_{\to \sigma_1^2} + \big(\underbrace{\tfrac{1}{N}(x_2 - \mu_2)\big)^2}_{\to \sigma_2^2} + \dots + \big(\tfrac{1}{N}(x_N - \mu_N)\big)^2$$

$$+ \frac{1}{N^2}(x_1 - \mu_1)(x_2 - \mu_2) + \dots \frac{1}{N^2}(x_1 - \mu_1)(x_N - \mu_N)$$

$$+ \frac{1}{N^2}(x_2 - \mu_2)(x_1 - \mu_1) + \dots \frac{1}{N^2}(x_2 - \mu_2)(x_N - \mu_N)$$

$$+ \dots$$

$$+ \frac{1}{N^2}(x_N - \mu_N)(x_1 - \mu_1) + \frac{1}{N^2}(x_N - \mu_N)(x_2 - \mu_2) + \dots \Big]$$

$\frac{1}{N}(x_1 - \mu_1)^2 = \sigma_1^2$ and so on

covariance terms are zero as assume

experiments done independently.

$$\Rightarrow \sigma_y^2 = \frac{1}{N^2}\left(\sigma_1^2 + \sigma_2^2 + \cdots \sigma_N^2\right)$$

$\hookrightarrow$ should this be there?