ELL301

24.04.2019

Problems:

uncertainty in Z

1. If $Z = 2x$, is $U(Z) = 2U(x)$?

or

$Z = x + x$

$\Rightarrow U(Z) = \sqrt{U^2(x) + U^2(x)} = \sqrt{2}\, U(x)$?

$$y = f(x_1, x_2, \ldots x_n)$$

$x_i = \mu_i \pm \sigma_i$ ← uncertainty

y = mean $\pm$ uncertainty in y?

Assumption 1

mean$(y) = f(\mu_1, \mu_2, \ldots \mu_n)$

Linearize around mean $(\mu_1, \mu_2, \ldots \mu_n, f(\mu_1, \mu_2 .. \mu_n))$

$$dy = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 + \ldots + \frac{\partial f}{\partial x_n} dx_n$$

$(dy)^2 \sim$ variance / uncertainty of y

$(dx_i)^2 \sim$ variance / uncertainty of x_i

$$(dy)^2 = \left(\frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 + \ldots + \frac{\partial f}{\partial x_n} dx_n \right)^2$$

$$= \left(\frac{\partial f}{\partial x_1}\right)^2 (dx_1)^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 (dx_2)^2 + \dots + \left(\frac{\partial f}{\partial x_n}\right)^2 (dx_n)^2$$

$$+ \frac{\partial f}{\partial x_1} \frac{\partial f}{\partial x_2} dx_1 . dx_2 + \text{other cross terms}$$

$$= \left(\frac{\partial f}{\partial x_1}\right)^2 (dx_1)^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 (dx_2)^2 + \dots + \left(\frac{\partial f}{\partial x_n}\right)^2 (dx_n)^2$$

Assumption 2 — if $dx_1 . dx_2 = 0$ on average

→ this assumes uncertainties in $x_1, x_2 \dots x_n$ are independent

$$(dy)^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 (dx_1)^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 (dx_2)^2 + \dots + \left(\frac{\partial f}{\partial x_n}\right)^2 (dx_n)^2$$

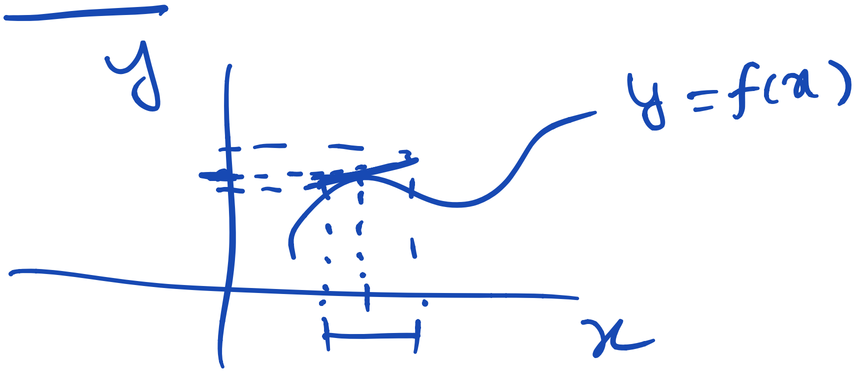
σ_y^2 or

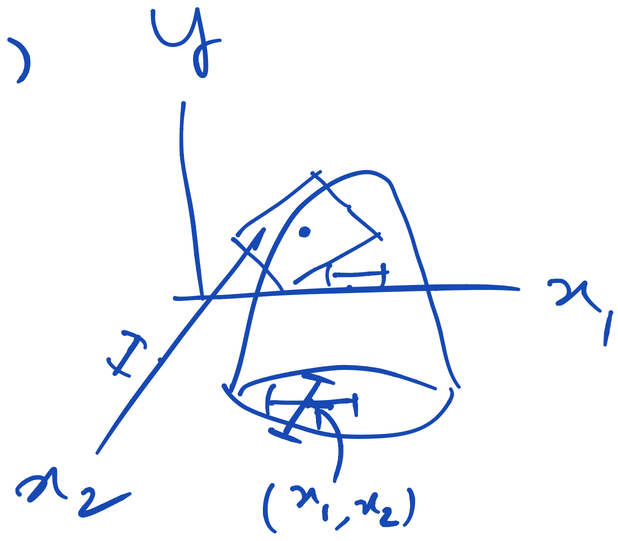
$$U_y^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 U_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 U_{x_2}^2 + \dots + \left(\frac{\partial f}{\partial x_n}\right)^2 U_{x_n}^2$$

uncertainty in y — uncertainty in x_1 — ... in x_2 — - - - in x_n

(worst-case is another way)

This is ~~how~~ one way to estimate uncertainty in y, when y is a function of $x_1, x_2 \dots x_n$ each of which have uncertainties.





Problems:

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<u>Ans</u>. $2U(x)$ as x & x are not independent, but same

2. $R_1 = 100 \pm 0.12$
   $R_2 = 50 \pm 0.08$

What is uncertainty in net resistance

- If connected in series,

$$R = R_1 + R_2 \rightarrow U_R = \sqrt{U_{R_1}^2 + U_{R_2}^2}$$

- If connected in parallel.

$$R = \frac{R_1 R_2}{R_1 + R_2} \rightarrow U_R = \sqrt{\left(\left.\frac{R_2^2}{(R_1+R_2)^2}\right|_{mean}\right)^2 U_{R_1}^2 + \left(\left.\frac{R_1^2}{(R_1+R_2)^2}\right|_{mean}\right)^2 U_{R_2}^2}$$

$$\frac{\partial (R)}{\partial R_1} = \frac{(R_1+R_2)R_2 - R_1 R_2}{(R_1+R_2)^2} = \frac{R_2^2}{(R_1+R_2)^2}$$

not 99, not 101   not 50.01, not 49.99

3. $R_1 = 100$, $R_2 = 50.00$ (both Ω)

With right number of significant figures

$R = R_1 + R_2 = 150$ (series)

150.00

R (parallel) $= \frac{R_1 R_2}{R_1 + R_2} =$ 33

33.33

33.3

minimum number of significant figures of 100, 50.00

$= \min\{3, 4\} = 3$

how many significant figures?

$R_1 \times R_2 = 5000$ (mathematical)

5000.000

50×10^2

50.00
100

0000
0 000x
5 0 00xx

5 0 0 0 . 0 0

50.00
10

0000
5 000x

5 0 0.0 0
50 ×10

→ 500× 10

4- length = 24.63 (4)
breadth = 1.001 (4)

area = what is expression upto significant figures.