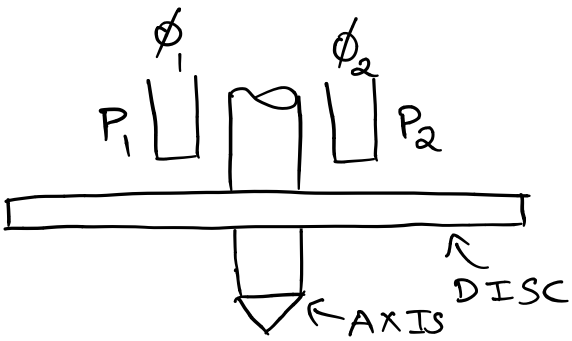
SOLUTIONS

ELL 301 MINOR TEST 2 DURATION = 1 HR MARKS = 20

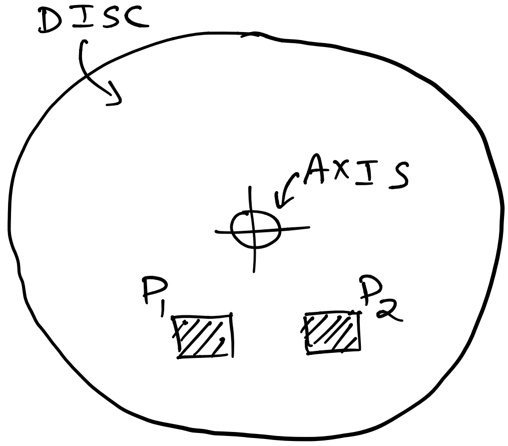
1. A thin aluminium disc is free to rotate about an axis through its centre. Two poles P_1 and P_2 produce alternating magnetic fluxes ϕ_1 and ϕ_2 cutting through the disc. If the disc is purely

$\phi_1 = \phi_m \sin\omega t$,

$\phi_2 = \phi_m \sin(\omega t - \beta)$



FRONT VIEW

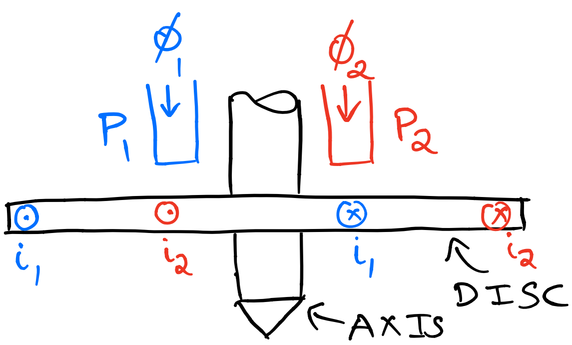


TOP VIEW

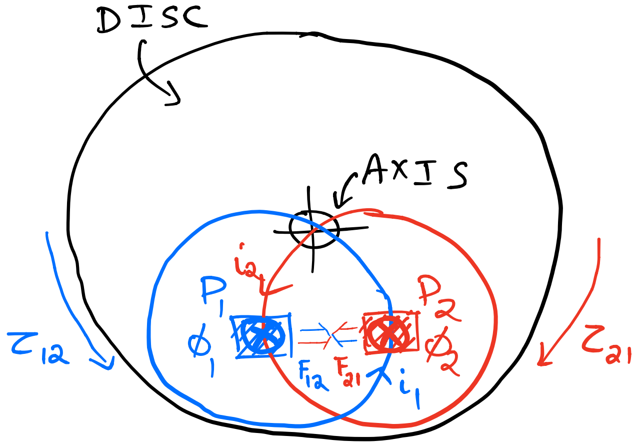
resistive, what is the torque acting on the disc? What is the torque if the disc has impedance $Z = |Z| e^{j\alpha}$? For what value of β and α is the torque maximum?

$\phi_1 = \phi_m \sin\omega t$,

$\phi_2 = \phi_m \sin(\omega t - \beta)$



FRONT VIEW



TOP VIEW

- Assuming flux directions at an instant as above, the currents induced in the disc may be obtained.

$$i_1 = \frac{\phi_m}{R}\,\omega\cos\omega t, \quad i_2 = \frac{\phi_m}{R}\,\omega\cos(\omega t-\beta) \qquad (1)$$

disc resistance

- Force act in indicated directions and produce torques.

$$F_{21} \sim \frac{\phi_m}{R}\,\omega\cos\omega t \times \phi_m \sin(\omega t-\beta) \longrightarrow \tau_{21} \qquad (1)$$

$$F_{12} \sim \frac{\phi_m}{R}\,\omega\cos(\omega t-\beta) \times \phi_m \sin\omega t \longrightarrow \tau_{12}$$

- Net torque, $\tau \sim \tau_{12} - \tau_{21}$

$$= \frac{\phi_m^2}{R}\,\omega\left[\cos(\omega t-\beta)\sin\omega t - \cos\omega t\sin(\omega t-\beta)\right]$$

$$= \frac{\phi_m^2\omega}{R}\,\sin\beta \qquad (1)$$

If disc has impedance $z = |z|\,e^{j\alpha}$

$$i_1 = \frac{\phi_m}{|z|}\,\omega\cos(\omega t-\alpha)$$

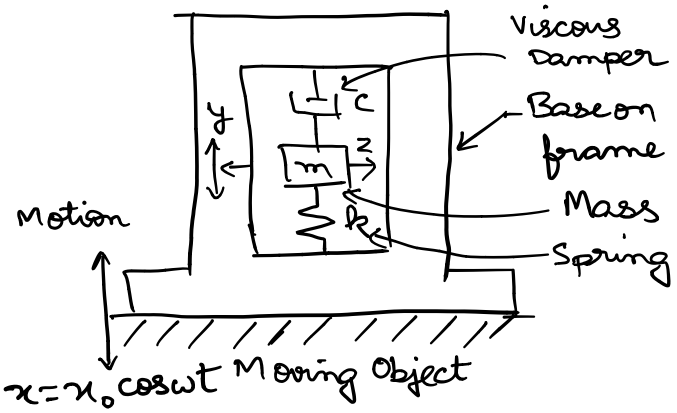
$$i_2 = \frac{\phi_m}{|z|}\,\omega\cos(\omega t-\beta-\alpha) \qquad (1)$$

$$\Rightarrow \quad \tau \sim \frac{\phi_m^2\,\omega}{|z|}\left[\cos(\omega t-\beta-\alpha)\sin\omega t - \cos(\omega t-\alpha)\sin(\omega t-\beta)\right]$$

$$= \frac{\phi_m^2\,\omega}{|z|}\,\sin\beta\cos\alpha \qquad (1)$$

Maximum when $\beta = 90^\circ$ (fluxes at right angles),
$\alpha = 0^\circ$ (purely resistive) (1)

2. A schematic diagram of a seismic transducer is



. The equation of motion of mass 'm' is,

$$m\ddot{y} = -c(\dot{y}-\dot{x}) - k(y-x).$$

For its motion, $z = y - x$, relative to the frame, this equation is $m\ddot{z} + c\dot{z} + kz = -m\ddot{x} = m\omega^2 x_0 \cos\omega t$.

a) Find the steady-state solution $z(t)$.

b) If Z_0 is the amplitude of $z(t)$, show that

$$\frac{Z_0}{x_0} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}, \quad r = \frac{\omega}{\omega_n}, \quad \omega_n = \sqrt{\frac{k}{m}}, \quad \xi = \frac{c}{2\sqrt{km}}$$

c) Find $\lim_{r\to\infty} \frac{Z_0}{x_0}$ (for constant ξ).

d) Find $\lim_{r\to 0} \frac{\omega_n^2 Z_0}{\omega^2 x_0}$ (for constant ξ).

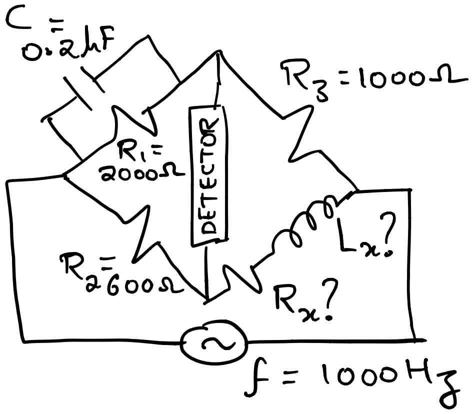
a) $z(t) = Z_0 \cos(\omega t - \phi)$ 1

$$Z_0 = \frac{m\omega^2 x_0}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}}, \quad \phi = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right)$$

b) $$\frac{Z_0}{x_0} = \frac{m\omega^2}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$ 1

c) $\lim_{r\to\infty} \frac{Z_0}{x_0} = 1 \Rightarrow$ for large enough r, $Z_0 \sim x_0$ 1

d) $\lim_{r\to 0} \frac{\omega_n^2}{\omega^2}\frac{Z_0}{x_0} = 1 \Rightarrow$ for small enough r, acceler- -ations are same 1

3. 

Determine the inductive impedance (R_x, L_x) connected in this balanced (Maxwell's) bridge circuit.

In balanced condition,

$$\frac{R_1 \parallel \frac{1}{j\omega C}}{R_2} = \frac{R_3}{\underbrace{R_x + j\omega L_x}_{Z_x}} \qquad (1)$$

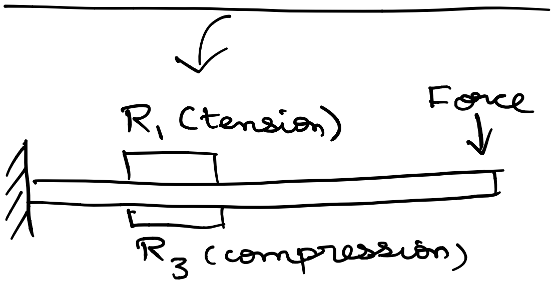
$$\Rightarrow \quad Z_x = \frac{R_2 R_3}{R_1} + j\omega R_2 R_3 C_1$$

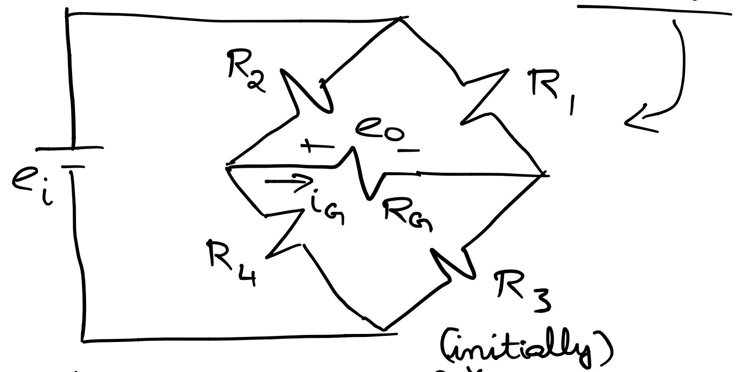
$$\Rightarrow \quad R_x = \frac{600 \times 1000}{2000} = 300\Omega$$

$$\& \Rightarrow \quad L_x = 600 \times 1000 \times 0.2 \times 10^{-6} \qquad (1)$$

$$= 120 \text{ mH}$$

4. Two electrical strain gauges are bonded to a Duralumin cantilever and connected in a bridge



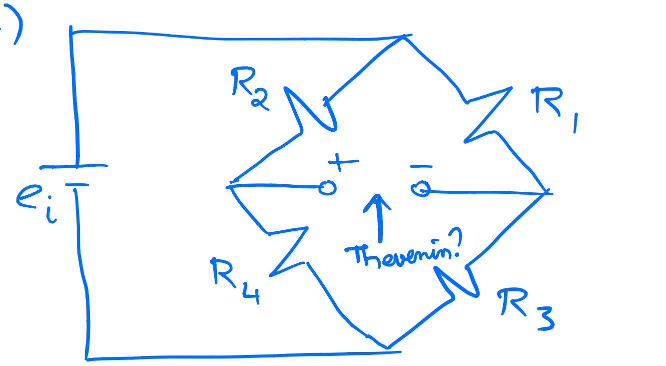


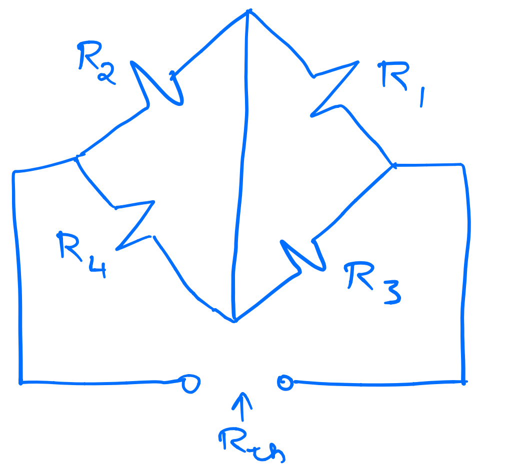
Each gauge has a resistance of $100\,\Omega$ and a gauge factor of 2.1. $R_2 = R_4 = 100\,\Omega$. $e_i = 4$ V. Stress is $200 \times 10^6\ N/m^2$. Young's modulus of Duralumin $70\ GN/m^2$.

a) If $R_G = 10\,k\Omega$, $e_o = ?$

b) Comment on the temperature compensation ability of this arrangement.

a)





$$V_{th} = e_i \left[\frac{R_4}{R_2 + R_4} - \frac{R_3}{R_1 + R_3} \right]$$

$$R_{th} = R_2 \| R_4 + R_1 \| R_3$$

$$Y = \frac{\text{stress}}{\text{strain}} \Rightarrow \text{Strain} = \frac{200 \times 10^6}{70 \times 10^9} = 2.86 \times 10^{-3} \quad 1$$

$$\Rightarrow \frac{\Delta R_1}{R_1} = 2.1 \times 2.86 \times 10^{-3} = 6 \times 10^{-3} = -\frac{\Delta R_3}{R_3}$$

$$\therefore\ R_1 = 100(1 + 6 \times 10^{-3}),\quad R_3 = 100(1 - 6 \times 10^{-3}) \quad 1$$

$$\Rightarrow V_{th} = 12\,mV,\quad R_{th} = 99.9982$$

$$\therefore\ e_o = 12\,mV \cdot \frac{10\,k\Omega}{10\,k\Omega + 99.9982} = 11.88\,mV$$ 1

(b) Arrangement is temperature compensated as the strain gauges are on adjacent arms.

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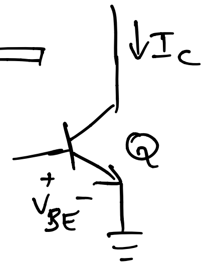
5. Properties of Silicon may be used to make a temperature sensor where a voltage is Proportional To Absolute Temperature.

a) Consider a BJT,



Assume, $I_c = I_s e^{qV_{BE}/kT}$

T: Absolute Temperature,

q: electron charge, k: Boltzmann constant,

$I_s = I_o e^{-qV_g/kT}$

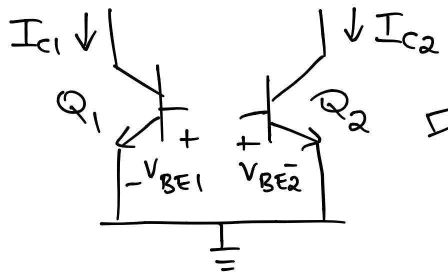
V_g: Silicon bandgap (temperature independent)

I_o: device parameter (assumed temperature independent)

$I_o \propto A$, area of transistor

Show that $V_{BE} = V_g + \frac{kT}{q} \ln \frac{I_s}{I_o}$, $\frac{\partial V_{BE}}{\partial T} = \frac{V_{BE} - V_g}{T}$

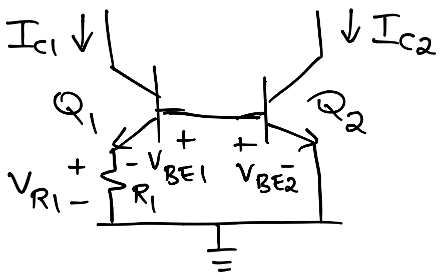
b) Consider two BJT's with $A_1 = 8A_2$ (area relation).



Show that $\Delta V_{BE} \overset{\text{defn.}}{=} V_{BE2} - V_{BE1}$

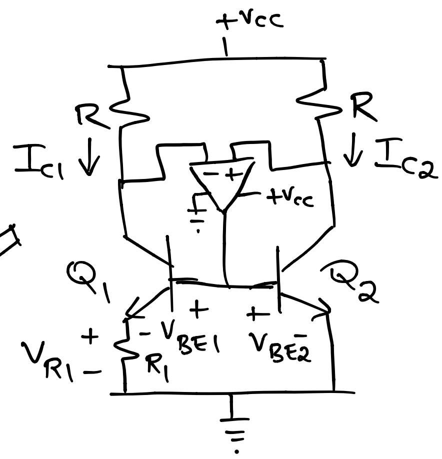
$= \frac{kT}{q} \ln\left(\frac{I_{c2}}{I_{c1}} \cdot 8\right)$

c)



Show that $V_{R1} = \Delta V_{BE}$.

d) Discuss whether the op-amp feedback can ensure $I_{c1} = I_{c2}$.



a) $I_c = I_s\, e^{qV_{BE}/kT}$

$= I_0\, e^{-qV_g/kT}\, e^{qV_{BE}/kT}$

$\Rightarrow \ln \dfrac{I_c}{I_0} = \dfrac{V_{BE}-V_g}{kT/q} \Rightarrow V_{BE} = V_g + \dfrac{kT}{q} \cdot \ln \dfrac{I_c}{I_0}$

$\dfrac{\partial V_{BE}}{\partial T} = \dfrac{k}{q} \ln \dfrac{I_c}{I_0} = \dfrac{V_{BE}-V_g}{T}$ 1

b) $\Delta V_{BE} \triangleq V_{BE2} - V_{BE1}$

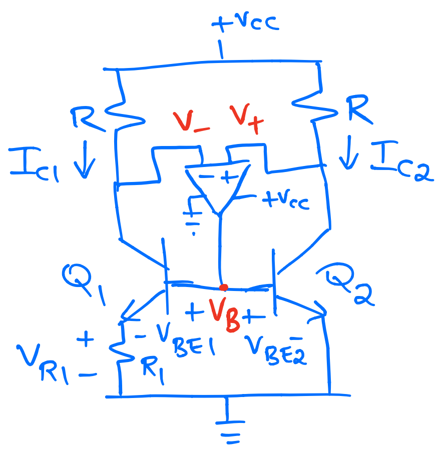
$= V_g + \dfrac{kT}{q} \ln \dfrac{I_{c2}}{I_{02}} - \left(V_g + \dfrac{kT}{q} \ln \dfrac{I_{c1}}{I_{01}} \right)$ 1

$= \dfrac{kT}{q} \ln \dfrac{I_{c2}}{I_{c1}} \cdot \dfrac{I_{01}}{I_{02}} = \dfrac{kT}{q} \ln \dfrac{I_{c2}}{I_{c1}} \cdot \dfrac{A_1}{A_2} = \dfrac{kT}{q} \ln \left(\dfrac{I_{c2}}{I_{c1}} \cdot 8 \right)$

c) $V_{BE2} = V_{BE1} + V_{R1}$

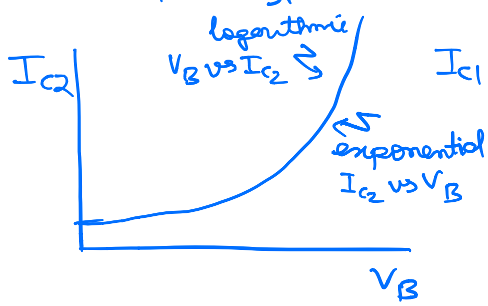
$\Rightarrow V_{R1} = V_{BE2} - V_{BE1} = \Delta V_{BE}$ 1

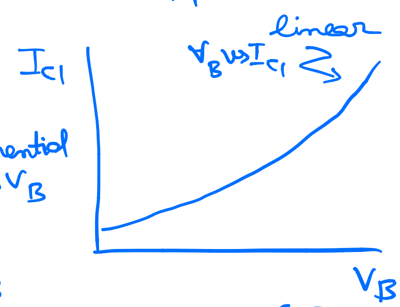
d)



$I_{c2} = I_{S2}\, e^{qV_{BE2}/kT} = I_{S2}\, e^{qV_B/kT}$

$I_{c1} = I_{S1}\, e^{qV_{BE1}/kT} = I_{S1}\, e^{\frac{q(V_B - I_{c1}R_1)}{kT}}$





If V_B is very small, I_{c1}, I_{c2} are small, $\dfrac{I_{c1}}{I_{c2}} = \underbrace{\dfrac{I_{S1}}{I_{S2}}}_{\approx 8} \cdot \underbrace{e^{-I_{c1}R_1 \cdot q/kT}}_{\approx 1}$

$\Rightarrow V_- < V_+ \Rightarrow V_B$ increases

If V_B is very large, I_{c1}, I_{c2} are large and $I_{c2} > I_{c1}$

$\Rightarrow V_+ < V_- \Rightarrow V_B$ decreases 1

$\therefore$ Can settle to a point where $I_{c1} = I_{c2}$

- For BJT, assume $I_C = I_S e^{V_{BE}/V_T}$, where I_c is collector current, V_{BE} is base-emitter voltage, $V_T = \frac{kT}{q}$ is thermal voltage, and I_S is a parameter that is
- process and temperature dependent $I_S = I_0 e^{-V_{G0}/V_T}$, I_0 is device parameter (temperature independent), V_{G0} is bandgap of silicon(,,). Show that

  $V_{BE} = V_{G0} + V_T \ln \frac{I_C}{I_0}$ $\qquad \frac{\partial V_{BE}}{\partial T} = \frac{k}{q} \ln \frac{I_C}{I_0} = \frac{V_{BE} - V_{G0}}{T}$

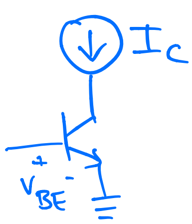
- For two BJTs show that $\Delta V_{BE} = V_{BE1} - V_{BE2} = V_T \ln \frac{I_{C1}}{I_{C2}} \cdot \frac{I_{S2}}{I_{S1}}$.

- Show that $V_{R1} = \Delta V_{BE}$ and $V_{R2} \propto V_T$

- Opamps.

The temperature dependence of silicon properties can be used to make temperature sensors.

- Consider a BJT, Assume $I_C = I_S e^{V_{BE}/V_T}$,



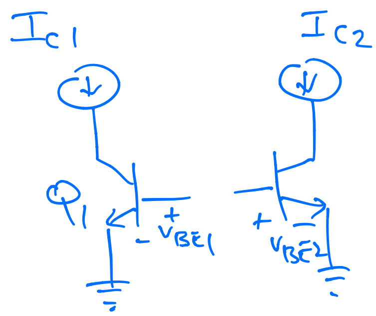
where I_C is collector current, V_{BE} is Base-Emitter voltage, $V_T = \frac{kT}{q}$ k is Boltzmann's constant, T is absolute temperature, q is charge of electron, $I_S = I_0 e^{-E_g/V_T}$, I_0 is a device parameter (temperature independent) $\propto$ area and E_g is bandgap of silicon (along ,,) Show that

$V_{BE} = V_{G0} + V_T \ln \frac{I_C}{I_0}$, $\frac{\partial V_{BE}}{\partial T} = \frac{V_{BE} - V_{G0}}{T}$.

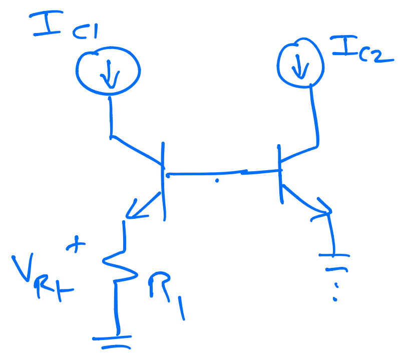
- Consider two BJTs and show that $\Delta V_{BE} = V_{BE1} - V_{BE2}$

$$= V_T \ln \frac{I_{C1}}{I_{C2}} \cdot \frac{I_{S2}}{I_{S1}}$$

$$= V_T \ln \frac{I_{C1}}{I_{C2}} \frac{I_{02}}{I_{01}}$$

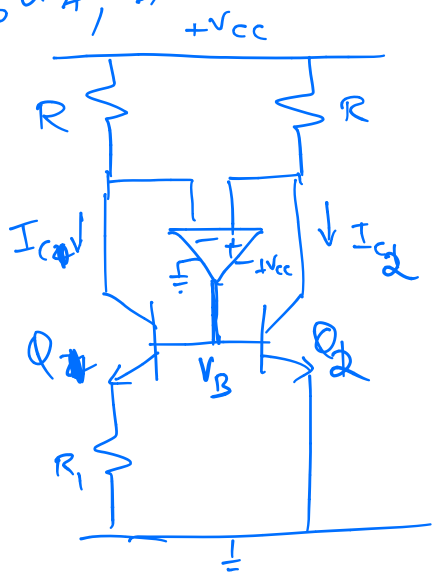


- Show that $V_{R_1} = \Delta V_{BE}$



- $I_0 \propto A_1 \Rightarrow$

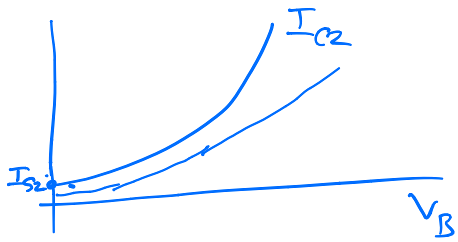
- Show that the op-amp feedback ensures $I_{C1} = I_{C2}$. Discuss whether if $A_1 = 8A_2$



$$I_{C1} = I_{S1} e^{\frac{V_B - I_{C1}R_1}{V_T}}$$

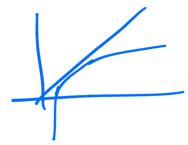
$$I_{C2} = I_{S2} e^{V_{BE2}/V_T}$$

$$= I_{S2} e^{V_B/V_T}$$



$V_B = V_T \ln \frac{I_{C1}}{I_{S1}} + I_{C1}R_1$

$V_B = V_T \ln \frac{I_{C1}}{I_{C2}}$



Suppose $I_{C1} \neq I_{C2}$.

$I_{C1} > I_{C2}$ or $I_{C1} < I_{C2}$

$\Rightarrow V_- < V_+$

$\Rightarrow V_B > 0$

$I_{C1} = I_{C2}$

$V_B = V_{CC}/2$

$I_{C1}\uparrow \Rightarrow V_-\downarrow \Rightarrow V_B\uparrow \Rightarrow I_{C2}\uparrow$

$I_{C1}\downarrow \Rightarrow V_-\uparrow \Rightarrow V_B\downarrow \Rightarrow I_{C2}\downarrow$

$I_{C2}\uparrow \Rightarrow V_+\downarrow \Rightarrow V_B\downarrow \Rightarrow I_{C2}\downarrow$

I_{C1}