SOLUTIONS

ELL 301 RE-MINOR TEST DURATION = 1 HR MARKS = 20

1. An experiment generates data as follows,

4 Marks

Input: $x_1, x_2, \ldots x_N$

$\downarrow \quad \downarrow \quad \quad \downarrow$

Output: $y_1, y_2, \ldots y_N$

It is desired to fit this to a straight line $y = Kx$. Determine the estimate of K by minimizing the cost $\sum_{i=1}^{N} (y_i - Kx_i)^2$. Discuss when this variance is the minimum variance achievable as per the Cramer-Rao Bound.

A

$$V(K) = \sum_{i=1}^{N} (y_i - Kx_i)^2$$

$$\frac{\partial V}{\partial k} = 0 \Rightarrow \sum_{i=1}^{N} (-2x_i)(y_i - Kx_i) = 0$$

$$\Rightarrow \hat{K} = \frac{\sum_{i=1}^{N} x_i y_i}{\sum_{i=1}^{N} x_i^2} \quad 2$$

$$\frac{\partial^2 V}{\partial k^2} = \sum_{i=1}^{N} x_i^2 \geq 0 \Rightarrow \text{minimum.} \quad 1$$

When $y = Kx + n$, and n is iid Gaussian noise, it achieves minimum variance as per the Cramer-Rao Bound. 1

2. Answer the following. Clearly show steps.

a) The radius of a sphere was estimated as (50 ± 0.5) mm. The estimated error in its mass is ___?___ %. <u>1 Mark</u>

b) Two resistances $R_1 = 100\ \Omega$ and $R_2 = 50.00\ \Omega$ are connected in parallel. Express the net resistance to the correct level of significant figures <u>2 Marks</u>

c) In the forthcoming redefinition of SI units, which fundamental constants are used to define standards for time, distance, mass, current, and temperature. <u>2 Marks</u>

Ans. a) $R = 50 \pm 0.5$ mm

$$\frac{\Delta R}{R} = \frac{0.5}{50} = 1\%$$

1

$$\text{mass} = \text{density} \times \text{volume} \sim R^3$$

$$\therefore \log(\text{mass}) = 3 \log R + \text{constant}$$

$$\Rightarrow \frac{\Delta\,\text{mass}}{\text{mass}} = 3\,\frac{\Delta R}{R} = 3\%$$

b) $$R_{\parallel} = \frac{R_1 R_2}{R_1 + R_2} = 33.333\ldots\ \Omega$$

(mathematically)

$$100 \begin{cases} \nearrow 101 \Rightarrow R_{11} = 33.4437\ldots \\ \searrow 99 \Rightarrow R_{11} = 33.2214\ldots \end{cases}$$

2

$$50.00 \begin{cases} \nearrow 50.01 \Rightarrow R_{11} = 33.3377\ldots \\ \searrow 49.99 \Rightarrow R_{11} = 33.3288\ldots \end{cases}$$

As first place after decimal is doubtful,

$R_{11} = 33.3\ \Omega$.

c) [time → Cs atom property]

distance → c, speed of light

current → e, charge of electron

mass → h, Planck's constant

temperature → k, Boltzmann's constant

2

3. A rectangular steel beam is used as a force measuring device by applying a force F at the free-end and correlating this with the deflection δ measured at the free-end by means of an LVDT. The dimensions of the beam are 4 Marks

length, $L = (50 \pm 0.5)$ mm,

breadth, $b = (8.20 \pm 0.1)$ mm, and

thickness, $t = (4.1 \pm 0.1)$ mm.

Determine the applied force F and its uncertainty if the deflection δ at the free-end was found to be (0.8 mm $\pm$ 2% of δ). Take the modulus of elasticity E of the beam as 2.07×10^{11} N/m^2 and use the following relationship for force and deflection at the free-end of the cantilever as $F = \frac{E\,b\,t^3}{4\,L^3}\,\delta$.

A. $F = \dfrac{2.07\times10^{11} \times 8.20\times10^{-3} \times (4.1\times10^{-3})^3}{4 \times (50\times10^{-3})^3} \times 0.8\times10^{-3}$

$= 0.0018717\ldots \times 10^5$ N

$= 187.17\ldots$ N

1

$$U_F = \sqrt{\left(\frac{\partial F}{\partial b}\right)^2 U_b^2 + \left(\frac{\partial F}{\partial t}\right)^2 U_t^2 + \left(\frac{\partial F}{\partial L}\right)^2 U_L^2 + \left(\frac{\partial F}{\partial \delta}\right)^2 U_\delta^2}$$

$$\frac{\partial F}{\partial b} = \frac{E t^3}{4 L^3}\delta = 22.83 \times 10^3, \quad U_b = 0.1 \times 10^{-3}$$

$$\frac{\partial F}{\partial t} = \frac{3 \cdot E b t^2}{4 L^3}\delta = 136.95 \times 10^3, \quad U_t = 0.1 \times 10^{-3}$$

$$\frac{\partial F}{\partial L} = -\frac{3 E b t^3}{4 L^4}\delta = 11.23 \times 10^3, \quad U_L = 0.5 \times 10^{-3}$$

$$\frac{\partial F}{\partial \delta} = \frac{E b t^3}{4 L^3} = 233.96 \times 10^3, \quad U_\delta = 0.016 \times 10^{-3}$$

$$\Rightarrow U_F = \sqrt{5.2 + 187.55 + 31.53 + (3.74)^2}$$

$$= 15.44$$

3

$$\therefore F = (187.17 \pm 8.2\%)\ N$$

4. A 230V, single phase energy meter has a constant load of 4 A passing through it for 6 hours at unity power factor. If the meter disc makes 2208 revolutions during this period, what is the meter constant ($\frac{\text{revolution}}{\text{kWhr}}$). Calculate the power factor of load if the number of revolutions made by the meter are 1472 when operating at 230V and 5A for 4 hours.

4 Marks

A Energy supplied $= V I \cos\theta \times t \times 10^{-3}$

$= 230 \times 4 \times 1 \times 6 \times 10^{-3}$

$= 5.52$ kWh

2

$\Rightarrow$ meter constant $= \frac{2208}{5.52} = 400$ rev/kWhr

Energy consumed when meter makes 1472 revolutions $= \frac{1472}{400} = 3.68$ kWh

2

As energy consumed $= V I \cos\theta \times t \times 10^{-3}$

$\Rightarrow \cos\theta = 0.8$.

5. Describe the functioning of an Analog to Digital Converter that works on the principle of staircase ramp compensation. Draw clear block diagram. 3 Marks