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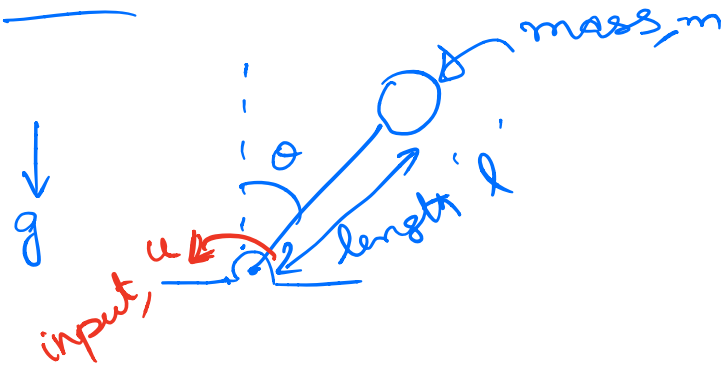
ELL333

How to develop a model of a bicycle? (or other system)

Starting points:

- Physical laws
- Trial and Error / System identification.



Newton's law

$\ddot{\theta}$ = some function of parameters $\{g, l, m\}$ and variables $\{\theta, \dot{\theta}\}$

Form of equation is important

$$\ddot{\theta} \pm \frac{g}{l} \sin\theta = u' \quad \{u' \propto u\}$$

Equally important is what the equation is in terms of, in this case θ and its derivative

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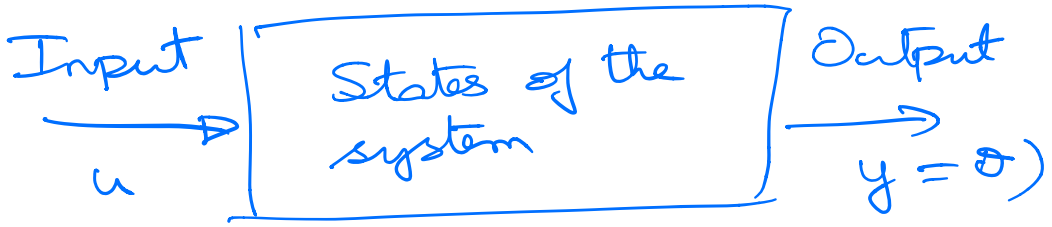

In most cases, identity of states is naturally given,

Mechanics / robotics → Position, velocity

Electrical → Current, voltage

Chemical Processes → Concentrations

How to define a state?

The state of a system is a set of variables that . . .

Bicycle model

$$(\quad)\ddot{q} + (\quad)\dot{q} + (\quad)q = f$$

$$q = \begin{bmatrix} \phi \\ \delta \end{bmatrix}$$

ϕ → balance angle

δ → steer angle

State-Space model (convention)

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

x: state, y: output, u: input

$n \times 1$ $\quad p \times 1$ $\quad m \times 1$

$A: n \times n \quad B: n \times m \quad C: p \times n \quad D: p \times m$

$$\ddot{\theta} + \frac{g}{l}\sin\theta = u$$

How to put this in the state-space form?

$$x_1 = \theta \qquad \dot{x}_1 = x_2$$
$$x_2 = \dot{\theta} \qquad \dot{x}_2 = -\frac{g}{l}\theta + u$$
$$= -\frac{g}{l}x_1 + u$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

How to handle $\sin\theta$?

$\sin\theta \approx \theta$ only about $\theta = 0$

→ $\ddot{\theta} + \frac{g}{l}\sin\theta = u$ (+, $-$?, $\sin\theta$, θ)

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -g/l & 0 \end{bmatrix}}_{A}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u$$

$$\ddot{\theta} + \theta = 0 \qquad \frac{g}{l} = 1$$

$$\Rightarrow \theta \sim \cos t, \sin t \qquad u = 0$$

$$\ddot{\theta} - \theta = 0$$

$$\Rightarrow \theta \sim e^{+t}, e^{-t}$$

$\therefore$ It should be minus sign