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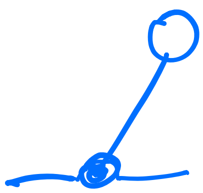
ELL333

definition of 'state'

The state of a system is the set (minimum?) of variables that, in the absence of external input, completely specifies its evolution over time, in accordance with governing physical laws.

$$\ddot{\theta} - \frac{g}{l}\sin\theta = 0 \qquad \underbrace{\overbrace{\theta(0),\ \dot{\theta}(0)}^{\text{initial conditions}}}_{\text{states}}$$



State-Space Form

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

We want to look at the solutions.

But why?

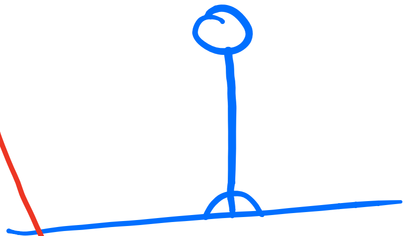
What can we get out of the solutions?

Bicycle example

Some things the solution might say

- standing bicycle, how fast does it fall?

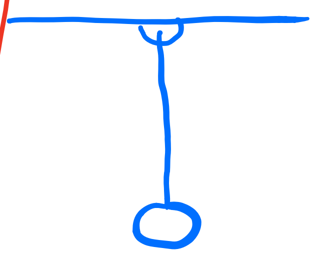
- if it were "exactly" standing?



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ +g/l & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$x_1 = \theta, \quad x_2 = \dot{\theta}$$

$$\theta(0) = 0, \quad \dot{\theta}(0) = 0$$



if initially vertical (inverted, or downward) and zero velocity, then the pendulum stays in that position (ideally)

Dynamics

- Stability : Inverted → unstable
  Downward → stable.

Notion of stability :
Stable if small perturbation to system from an initial state decays back to the initial state as time increase.

long-term behaviour

To understand dynamics and stability we are looking for solutions of this,

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Focus here, simplest case

$$\dot{x} = Ax$$

Propose a solution, $x(t) = e^{At} \cdot c$

(e^{At}: matrix exponential $(n \times n)$ matrix; c: $n \times 1$ vector constant)

definition

$$e^{At} \triangleq I + tA + \frac{t^2}{2!}A^2 + \dots + \frac{t^k}{k!}A^k + \dots$$

$$LHS = \dot{x} = \frac{d}{dt}x(t) = \frac{d}{dt}e^{At}.c$$

$$= \frac{d}{dt}\left[I + tA + \frac{t^2}{2!}A^2 + \dots \frac{t^k}{k!}A^k + \dots\right]c$$

$$= \left[0 \quad A + tA^2 + \dots + \frac{t^{k-1}}{(k-1)!}A^k + \dots\right]c$$

$$= A \cdot \underbrace{\left[I + tA + \dots \frac{t^{k-1}}{(k-1)!}A^{k-1} + \dots\right]c}_{x(t)} = RHS$$

$\therefore \ x(t) = e^{At} \cdot c$ is a solution.

Three questions

1. Solution exists. Is it unique? → Yes, because of existence and uniqueness theorems of differential equations.
2. How to determine c? Put $t=0$, $x(0) = e^{A0} \cdot c$
3. How to compute e^{At}?

→ $e^{A0} = I$ + everything else is zero

$\therefore \ x(t) = e^{At} \cdot x(0)$

#1 compute

→ $(sI - A)^{-1} \longrightarrow \mathcal{L}^{-1} \longrightarrow e^{At}$

[implicit that $(sI-A)^{-1}$ is the Laplace Transform of e^{At}]

examples

$A = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}$, $A = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

Definition of ‘state’

ELL333 Multivariable Control

Friedland, Pg 16

The state of a dynamic system is a set of physical quantities, the specification of which (in the absence of external excitation) completely determines the evolution of the system.

Astrom & Murray, Pg 40

One of the triumphs of Newton's mechanics was the observation that the motion of the planets could be predicted based on the current positions and velocities of all planets. It was not necessary to know the past motion. The *state* of a dynamical system is a collection of variables that completely characterizes the motion of a system for the purpose of predicting future motion. For a system of planets the state is simply the positions and the velocities of the planets. We call the set of all possible states the *state space*.

Polderman & Willems, Pg 115

State variables either show up naturally in the modeling process or they can be artificially introduced. State variables have the property that they parametrize the *memory* of the system, i.e., that they "split" the past and future of the behavior.

A. Bensoussan, Pg 1-2

A key element is the **state** representation of dynamical systems, also called the **internal** representation.

The internal representation introduces the very important concept of *state of the system*. The idea is reminiscent of that of a *knowledge model*, in which one models as accurately as possible all the components of the system, starting with the physical laws that are involved. For instance, if a dynamical system is a rocket in flight, then one will use the laws of mechanics and possibly more advanced physical and chemical laws to describe the propulsion. The state will be the position and velocity of the center of the rocket, and there may be additional state variables.