ELL 333
06.08.2019

What does input (u) and output (y) have use for?

- to change state
- to control state

using 'u'

- to observe state

using y

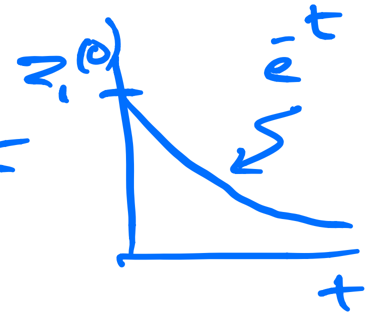
$$\dot{z}_1 = -z_1 + u$$
$$\dot{z}_2 = -2z_2$$
$$\dot{z}_3 = -3z_3 + u$$
$$\dot{z}_4 = -4z_4$$

$$y = z_1 + z_2$$

In a bicycle example, what do we want the inputs (steering, shifting weight) to do?

$$\dot{z}_1 = -z_1 + u$$

If $u=0$, $z_1(t) = z_1(0)\, e^{-t}$



What can we use 'u' for?

- change the state?

Suppose want $z_1 = 10 \neq 0$?

$u = 10$

$\dot{z}_1 = -z_1 + 10$

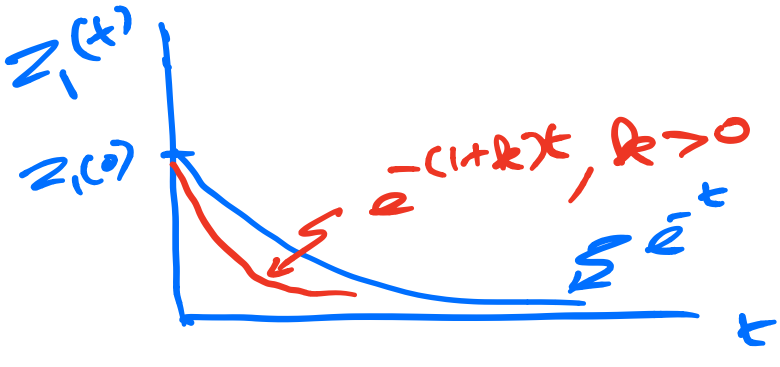
$z_1(t) = z_1(0)e^{-t} + 10(1-e^{-t})$

also in bicycle, may want it not to be always vertical, such as when making a turn.

- $u = -k z_1$?

$\dot{z}_1 = -z + (-k z_1)$

$z_1(t) = z_1(0)\, e^{-(1+k)t}$



$\therefore$ Using $u = -k z_1$, we can make dynamics slow $\longleftrightarrow$ fast

stable $\longleftrightarrow$ unstable

At least two things, we want to use input (u) to control
1. Change the state
2. shape the dynamics

Things we did	Dynamics & Stability	Controllability
• Notion / Definition	Lyapunov stability (small perturbations should decay)	?
• Solution	$\dot{x} = Ax$ $\Rightarrow x(t) = e^{At} \cdot x(0)$	$\dot{x} = Ax + Bu$ $\Rightarrow x(t) = ?$
• Co-ordinate transformation	Diagonalization $A = UDU^{-1}$	Companion form $A = VCV^{-1}$
• Test	$Re\{eig\{A\}\} < 0$ for decay	$rank[B \; AB \ldots A^{n-1}B] = n$

Where do "uncontrollable" systems arise?

→ mathematical example in last lecture (from last major)

→ more physical examples in Friedland

These are some reasons that we study controllability.

Solution of $\dot{x} = Ax + Bu$

$$x(t) = e^{At} c(t)$$

searching for particular solution from homogeneous solution

$$\Rightarrow \dot{x} = \frac{d}{dt} e^{At} \cdot c(t)$$

$$= \left(\frac{d}{dt} e^{At}\right) c(t) + e^{At} \cdot \frac{d}{dt} c(t)$$

$$= A e^{At} c(t) + e^{At} \frac{d}{dt} c(t)$$

$$\Rightarrow \quad = A \underbrace{e^{A(t)} c(t)}_{x} + Bu$$

$$\Rightarrow \cancel{A e^{At} c(t)} + e^{At} \frac{d}{dt} c(t) = \cancel{A e^{At} c(t)} + Bu$$

$$\Rightarrow e^{At} \frac{d}{dt} c(t) = Bu$$