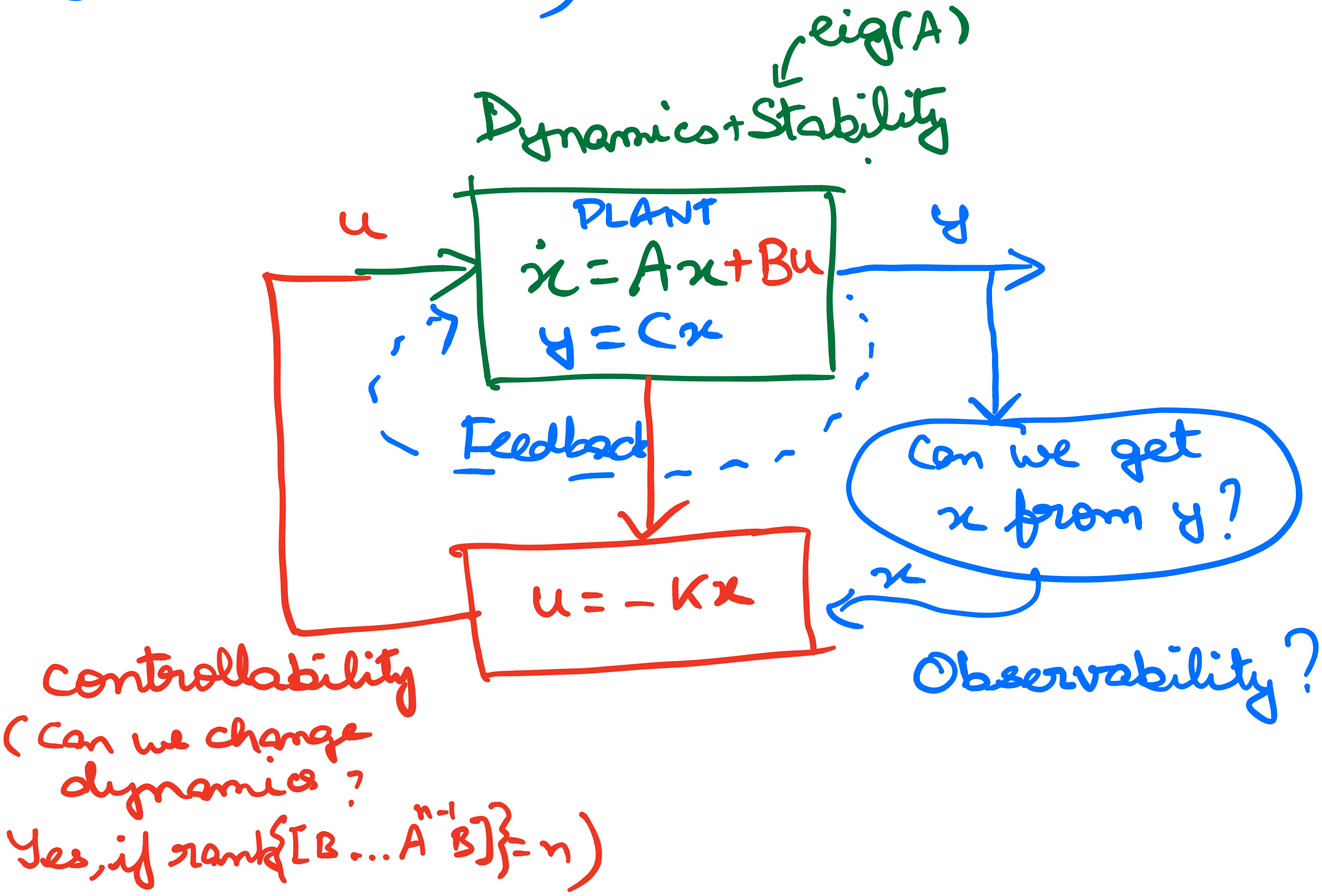
ELL333

05.09.2019



* Output feedback may also be considered

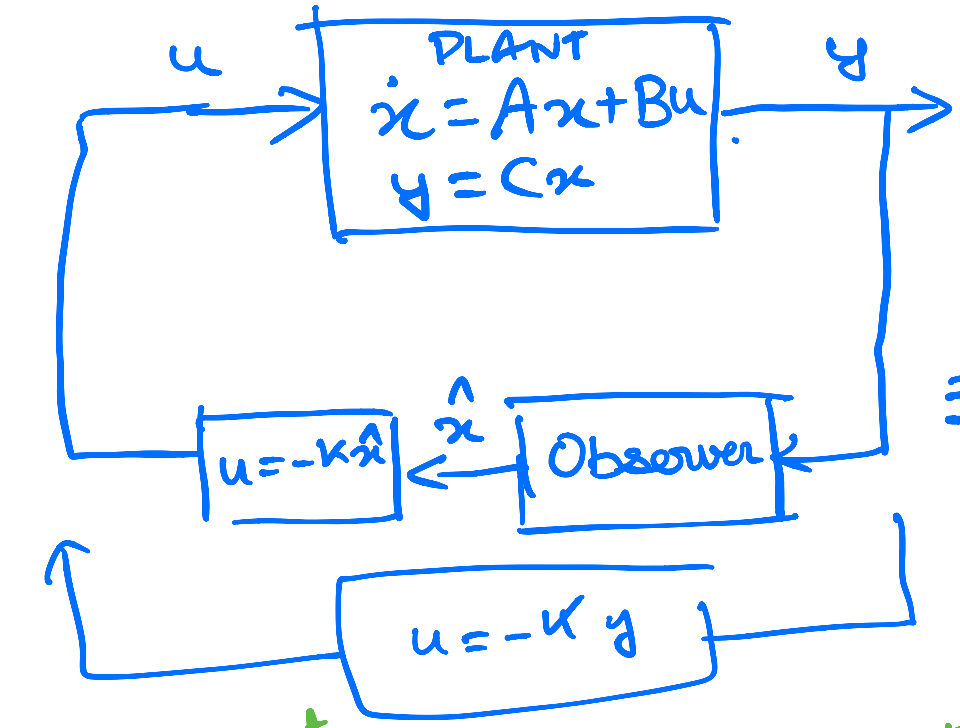
$$u = -Ky$$

$$\Rightarrow \dot{x} = Ax + Bu \qquad (u = -Ky = -KCx)$$

$$y = Cx$$

$$\Rightarrow \dot{x} = \underbrace{(A - BKC)}_{\text{How to change eigenvalues of this?}} x$$

Standard view



$\equiv$ State Estimation + state feedback

$\equiv$ Output feedback

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measurement

what we want to estimate

$$y = C x + \text{Noise}$$

$p \times 1$ $\quad$ $p \times n$ $\quad$ $n \times 1$

Measurement model (continuous time)

Can also have measurement in discrete time

$$y_k = C x_k + \text{Noise}_k$$

All state equation so far have been continuous.

May also have discrete

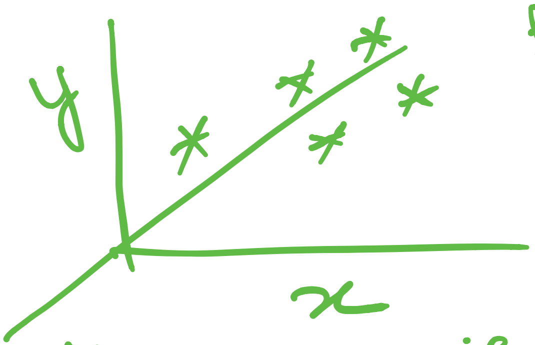
$$x_{k+1} = A x_k + B u_k$$

where could this arise
- server packets
- time-series finances

How to estimate x from y?

$$y = Cx + \text{Noise}$$

$$y_k = Cx_k + \text{Noise}_k$$



$y = mx + \text{noise}$

Find slope of line by minimizing $\sum_{i=1}^{N} (y_i - m x_i)^2$ over m to find what $\rightarrow$ estimate $\hat{m}$?

What's similarity/difference with state estimation?

State estimation

Observer

Kalman Filter

In the deterministic limit, we try to find the estimate of state from output

Finding the best estimate of state by minimizing some cost

Optimum observer

Things we did	Dynamics & Stability	Controllability	Observability
• Notion / Definition	Lyapunov stability (small perturbations should decay)	?	?
• Solution	$\dot{x} = Ax$ $\Rightarrow x(t) = e^{At} \cdot x(0)$	$\dot{x} = Ax + Bu$ $\Rightarrow x(t) = ?$	$\dot{x} = Ax$ $y = Cx$ $\Rightarrow y = ?$ $y(t) = C e^{At} x(0)$
• Co-ordinate transformation	Diagonalization $A = UDU^{-1}$	Companion form $A = VCV^{-1}$	Companion form $A = VOV^{-1}$
• Test	$\mathrm{Re}\{\mathrm{eig}\{A\}\} < 0$ for decay	$\mathrm{rank}[B \; AB \ldots A^{n-1}B] = n$	$\mathrm{rank}\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} = n$
• Design		eigenvalue assignment	observer design

$$\dot{x} = Ax$$
$$y = Cx$$

(we'll assume no input for now, $u = 0$)

How to get x from y?

Possible answers

- if $C = I$, then $x = y$
- if C is invertible (square), then $x = C^{-1} y$

- if C is not full rank, then?
  (covers if C is square or not square)

$C' \times \Sigma y = C x$

transpose

$\Rightarrow \quad C' y = C' C x$

$\Rightarrow \quad x = (C'C)^{-1} C' y$, if $C'C$ is invertible

$(C'C)^{-1} . C'$ generalized inverse
pseudo inverse
Moore Penrose

Is $(C'C)$ invertible
Suppose $p = n$, $\text{rank}(C) < n$

What $\text{rank}(C')$?

Also less than n, as $\text{rank}(C) = \text{rank}(C')$

What about $\text{rank}(C'C) \leq \text{rank}(C) < n$