ELL 333

17.09.2019

If it can be done! Answered by checking rank of $\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$?

How to get state from output observations ?

$$\dot{x} = Ax$$
$$y = Cx$$

- Invert C, if it is invertible

- $y = Cx \Rightarrow C' y = C' C x$
  $\Rightarrow x = (C'C)^{-1} C' y$

example of observable system, but $(C'C)^{-1}$ doesn't exist.

$C = [1 \quad 0]$, $C' = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow C'C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, which is not invertible

Suppose

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(if it was $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then x_2 is not connected to output, but in what is supposed it is)

$$\Rightarrow \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

which has rank 2.

$\dot{x} = Ax$

$y = Cx$

$\hat{x}$: state estimate

Suppose we want to estimate x from y, what do we need?

- y is given
- C?
- A?

To design controller, we needed x so that the input $u = -Kx$

We want $\hat{x} = x$

define error $e = x - \hat{x}$ we don't know this

we want error to be zero.

Agree: $\hat{x}$ changes with time.
(as x changes with time and want $\hat{x} \to x$)

How should $\hat{x}$ change with time?

* $\dot{\hat{x}} = A\hat{x}$

$$e = x - \hat{x} \Rightarrow \dot{e} = \dot{x} - \dot{\hat{x}}$$
$$= Ax - A\hat{x}$$
$$= Ae$$

$\Rightarrow e(t) = \exp(At) . e(0)$

$\Rightarrow e \to 0$ if A has eigenvalues with negative real parts

- $\dot{\hat{x}} = A\hat{x} + \underbrace{Ly}_{\text{adding in a "linear" way}}$ adding output, because that's what we want to use to estimate x

A constant

$e = x - \hat{x}$

$\Rightarrow \dot{e} = ? = \dot{x} - \dot{\hat{x}}$

$= Ax - A\hat{x} - LCx$

$= Ax - A(x-e) - LCx$

$= Ae - LCx$

$\dot{x} = Ax + Bu$

- $\dot{\hat{x}} = \hat{A}\hat{x} + Ly + \hat{B}u$ (Replaced A with $\hat{A}$, may get $\hat{A} = A$?

$\Rightarrow \dot{e} = ? =$

$\dot{e} = \dot{x} - \dot{\hat{x}}$

$= Ax - \hat{A}\hat{x} - Ly + Bu - \hat{B}u$

$= Ax - \hat{A}(x-e) - LCx + (B-\hat{B})u$

$= \hat{A}e + (A - \hat{A} - LC)x + (B-\hat{B})u$

We don't know x, so we set

$A - \hat{A} - LC = 0$

$\Rightarrow \hat{A} = A - LC$

$\Rightarrow \dot{e} = (A - LC)e$

set $\hat{B} = B$

Q. When does $e(t) \to 0$?

A. when eigenvalues of $(A-LC)$ have negative real part

This is the observer.

$$\dot{\hat{x}} = (A-LC)\hat{x} + Ly$$

[add $+Bu$ if required]

This gives error equation $\dot{e} = (A-LC)e$ and whether $e \to 0$ depends on choice of eigenvalues of $A-LC$, which we have to still talk about

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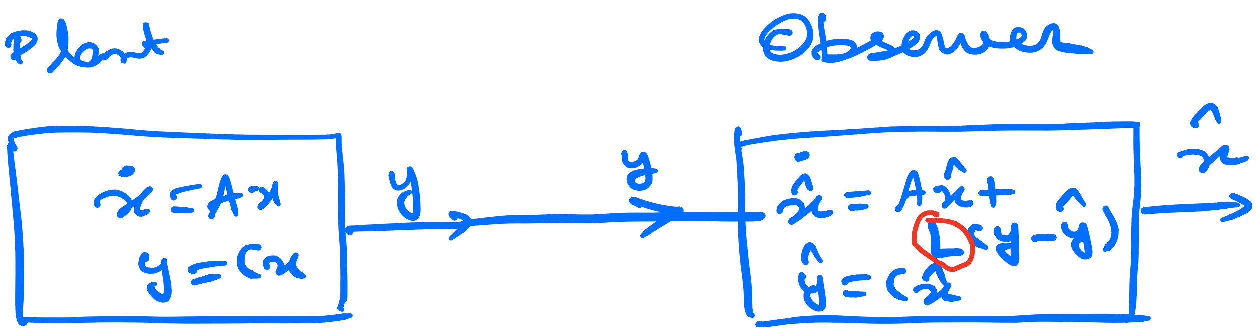
$$\dot{\hat{x}} = A\hat{x} - LC\hat{x} + L\overbrace{Cx}^{y}$$

$$= A\hat{x} + L(\underbrace{Cx}_{y} - \underbrace{C\hat{x}}_{\triangleq \hat{y}})$$

$$\dot{\hat{x}} = A\hat{x} + L(y - \hat{y})$$

actual output

estimated output.



How do we get L so that eigenvalues of $A - LC$ have negative real parts?

What did we do in controller design?

$$Q_c = [B \quad AB \quad \dots \quad A^{n-1}B] \to \text{check controllability}$$

$$Q_o = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \to \text{check observability}$$

$$u = -Kx \Rightarrow \dot{x} = Ax + Bu = Ax - BKx = \underline{\underline{(A-BK)}}x$$

We have done eigenvalue assignment for $A-BK$? How is it related to eigenvalue assignment of $A-LC$?

$$\boxed{\underset{n\times n}{A} - \underset{n\times m}{B}\underset{m\times n}{K}} \qquad \boxed{\underset{n\times n}{A} - \underset{n\times p}{L}\underset{p\times n}{C}}$$

Suppose known how to do eigenvalue assignment? $\longrightarrow$ Does it help here?