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ELL333

Observability	$=$ (Controllability)^T
$\text{rank}\left\{\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1}\end{bmatrix}\right\} = n$	$\text{rank}\{[B \; AB \ldots A^{n-1}B]\} = n$
algebraic tests	
$\dot{x} = Ax$, $y = Cx$; want $\hat{x} - x \to 0$; $\dot{\hat{x}} = A\hat{x} + \underbrace{L}_{n\times p}(y - C\hat{x})$ — observer	$\underbrace{u}_{m\times 1} = -\underbrace{K}_{m\times n}\underbrace{x}_{n\times 1}$; $\dot{x} = Ax + Bu$ — controller
Design L to place eigenvalues of $\underline{A - LC}$	Designed K to place eigenvalues of $\underline{A - BK}$

? $e = x - \hat{x}$

$\dot{e} = \dot{x} - \dot{\hat{x}}$

$= Ax - A\hat{x} - L(Cx - C\hat{x})$

$= Ae - LCe$

$= (A - LC)e$

?

?

$u = -Kx$

$\dot{x} = (A - BK)x$

this was done before.

~~How~~

Can we use K-design to place eigenvalues of A-BK for L-design to place eigenvalues of A-LC?

Ans $\to$ Transpose

Identify as follows

$$(A - LC)^T = \begin{array}{ccc} A^T & - & C^T \, L^T \\ ||| & & ||| \quad ||| \\ A & - & B \quad K \end{array}$$

(A, B) is controllable $\Leftrightarrow$ can choose K to place eigenvalues of $A - BK$.

What do we need to place eigenvalues of $A^T - C^T L^T$ by choosing L^T

$\Leftrightarrow$ (A^T, C^T) is controllable

$\Leftrightarrow$ (A, C) is observable

Example: $\dot{x} = Ax, \; y = Cx$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Design observer for this so that $\hat{x} \to x$?

1. Check observability i.e. rank$\left\{\begin{bmatrix} C \\ CA \end{bmatrix}\right\} = 2$.

2. $\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x})$ Choose L to place eigenvalues of $A - LC$

Quiz 6

If (A, C) is observable, show that

$$\operatorname{rank}\left\{ \begin{array}{cc} \begin{bmatrix} \lambda I - A \\ C \end{bmatrix} & \begin{array}{l} n \times n \\ p \times n \end{array} \end{array} \right\} = n$$

for all complex numbers λ.