ELL333
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AM 8.5

$$J \frac{d^2\phi}{dt^2} - \frac{D v_0}{b}\frac{d\delta}{dt} = mgh\phi + \frac{m v_0^2 h}{b}\delta$$

ϕ – tilt angle
δ – steering angle

Check observability? rank $\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} = n$

Find C

$$\underset{\text{output}}{y} = \phi \qquad \overset{C}{[1\ 0\ 0\ 0]}$$

$$\delta \qquad [0\ 0\ 1\ 0]$$

$$\begin{bmatrix} \phi \\ \delta \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

What could be possible states?

$$\begin{bmatrix} \phi \\ \dot\phi \\ \delta \\ \dot\delta \end{bmatrix} \checkmark$$

We need A?

$\rightarrow$ problem because $\frac{d^2\delta}{dt^2}$ not there.

We can try . . . .

$$J\frac{d^2\phi}{dt^2} - \frac{DV_0}{b}\frac{d\delta}{dt} = mgh\phi + \frac{mV_0^2h}{b}\delta$$

Assume δ is known, input u

$$J\frac{d^2\phi}{dt^2} - mgh\phi = \overbrace{\frac{DV_0}{b}\frac{d\delta}{dt} + \frac{mV_0^2h}{b}\delta}^{u}$$

$$\underline{x} = \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix}$$

$$\underline{C} \quad [1 \quad 0], \; [0 \quad 1], \; \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{A} \quad \begin{bmatrix} 0 & 1 \\ \frac{mgh}{J} & 0 \end{bmatrix}$$

observable

$$\text{rank}\left\{\begin{bmatrix} C \\ CA \end{bmatrix}\right\} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ \frac{mgh}{J} & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ \frac{mgh}{J} & 0 \end{bmatrix}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$2 \qquad\qquad 2 \qquad\qquad 2$$

Observable in all three cases

Design an observer?

- Observer equations
- Find the unknown gain matrix L
- $\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x})$
- Find L such that eigenvalues of $A-LC$ are in left half complex plane.

Quiz 7

$\dot{x} = Ax$
$y = Cx$

Design observer for

$$A = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

and place poles at roots of

i.e eigenvalues of observer

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0, \quad \omega_0 = 10, \zeta = 0.6.$$

i.e. $s^2 + 12s + 100 = 0$

roots are $\dfrac{-12 \pm \sqrt{144 - 400}}{2}$

$$= \frac{-12 \pm j16}{2} = -6 \pm j8$$

$$\begin{array}{r} 400 \\ 144 \\ \hline 256 \end{array}$$