

ELL333  
20.09.2019

AM 8.5

$$J \frac{d^2 \phi}{dt^2} - \frac{D v_0}{b} \frac{d\delta}{dt} = mgh\phi + \frac{m v_0^2 h \delta}{b}$$

$\phi$  - tilt angle  
 $\delta$  - steering angle

Check observability?  $\text{rank} \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} = n$

Find C

$$\begin{array}{l} \text{output} \rightarrow y = \begin{bmatrix} \phi \\ \delta \end{bmatrix} \end{array} \quad \begin{array}{l} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{array} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

What could be possible states?

$$\begin{bmatrix} \phi \\ \dot{\phi} \\ \delta \\ \dot{\delta} \end{bmatrix}$$

We need A?

→ problem because  $\frac{d^2 \epsilon}{dt^2}$  not there.

We can know ...

$$J \frac{d^2 \phi}{dt^2} - \frac{DV_0}{b} \frac{dS}{dt} = mgh\phi + \frac{mV_0^2 h}{b} S$$

Assume S is known, input u

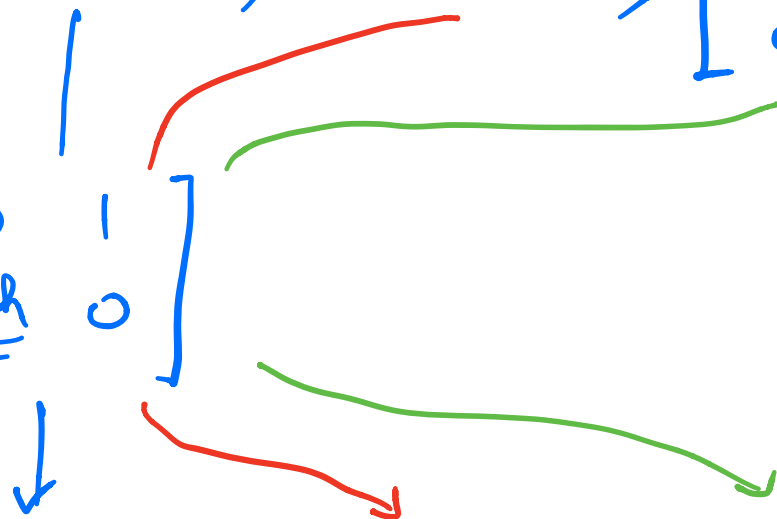
$$J \frac{d^2 \phi}{dt^2} - mgh\phi = \underbrace{\frac{DV_0}{b} \frac{dS}{dt} + \frac{mV_0^2 h}{b} S}_u$$

$$\underline{x} = \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix}$$

$$\underline{C} \quad [1 \ 0], [0 \ 1], [1 \ 0]$$

$$\underline{A} \quad \begin{bmatrix} 0 & 1 \\ \frac{mgh}{J} & 0 \end{bmatrix}$$

observable



$$\text{rank} \begin{Bmatrix} C \\ CA \end{Bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \vdots \begin{bmatrix} 0 & 1 \\ \frac{mg}{J} & 0 \end{bmatrix} \vdots \begin{bmatrix} c & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow$   
 $2 \quad \quad \quad 2 \quad \quad \quad 2$

Observable in all three cases

Design an observer?

- ↳ Observer equations
- ↳ Find the unknown gain matrix L
- ↳  $\dot{\hat{x}} = A \hat{x} + L (y - C \hat{x})$
- ↳ Find L such that eigenvalues of  $A - LC$  are in left half complex plane.

# Quiz 7

$$\dot{x} = Ax$$
$$y = Cx$$

Design observer for

$$A = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}, \quad C = [0 \quad 1]$$

and place poles <sup>i.e. eigenvalues of observer</sup> at roots of  $s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$ ,  $\omega_0 = 10$ ,  $\zeta = 0.6$ .

i.e.  $s^2 + 12s + 100 = 0$

roots are  $\frac{-12 \pm \sqrt{144 - 400}}{2}$

$$= \frac{-12 \pm j16}{2} = -6 \pm j8$$

$$\begin{array}{r} 400 \\ 144 \\ \hline 256 \end{array}$$