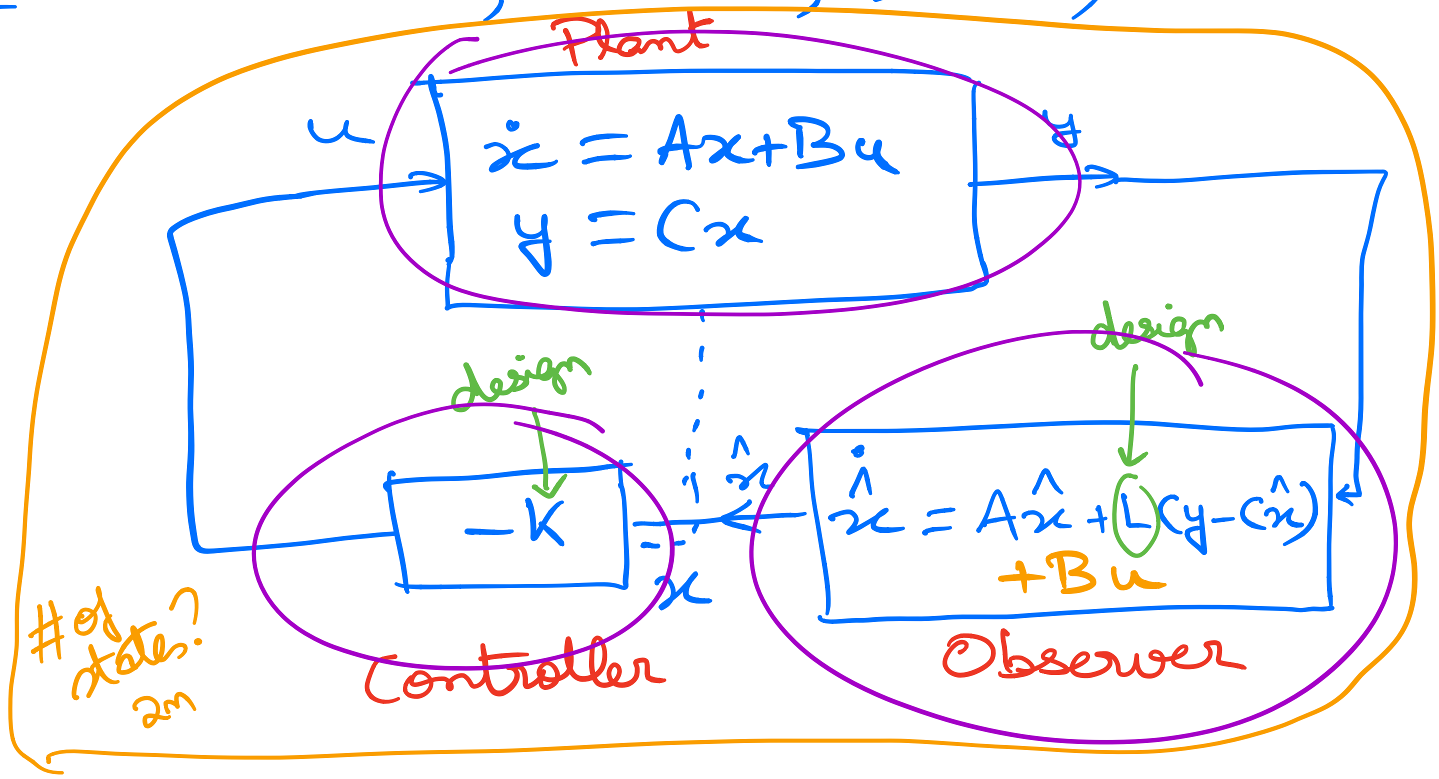
24.09.2019

ELL33

Minor Test 2, 4-5PM, LH108, 25.09.2019



Specification: Closed-loop system should have desired eigenvalues.

Controller design

$u = -Kx$ to place eigenvalues of $A-BK$

Observer design

chose L to place eigenvalues of $A-LC$

When we combine all these parts, then what is the eigenvalues of overall system, to get the dynamics?

$$\frac{d}{dt}\begin{bmatrix} x \\ \hat{x} \end{bmatrix}_{2n\times 1} = ?$$

$$\dot{x} = Ax + Bu, \quad y = Cx$$
$$\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x}), \quad u = -K\hat{x}$$

does this affect as initially designed for $u = -Kx$

$$\dot{x} = Ax - BK\hat{x}$$
$$\dot{\hat{x}} = A\hat{x} + LCx - LC\hat{x}$$
$$= LCx + (A - LC)\hat{x}$$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \underbrace{\begin{bmatrix} A & -BK \\ LC & A - LC \end{bmatrix}}_{2n \times 2n} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

How to determine eigenvalues of ( )?

$$e = x - \hat{x}$$
$$\dot{e} = \dot{x} - \dot{\hat{x}}$$
$$= Ax - BK\hat{x} - LCx - (A - LC)\hat{x}$$
$$= (A - LC)x - BK\hat{x} - (A - LC)\hat{x}$$
$$= (A - LC)e - BK\hat{x}$$

extra term

Redefine observer
$$\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x}) + Bu$$

$$\dot{x} = Ax + Bu, \quad y = Cx$$
$$\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x}) + Bu, \quad u = -K\hat{x}$$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \underbrace{\begin{bmatrix} A & -BK \\ LC & A-LC-BK \end{bmatrix}}_{\text{eigenvalues?}} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

$$e = x - \hat{x}$$

$$\dot{e} = \dot{x} - \dot{\hat{x}} = Ae - L(Cx - C\hat{x})$$
$$= (A-LC)\,e$$

$$(x, \hat{x}) \xrightarrow[\text{mapping}]{\text{invertible}} (x, e)$$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \left[\begin{array}{c|c} A-BK & BK \\ \hline 0 & A-LC \end{array}\right] \begin{bmatrix} x \\ e \end{bmatrix}$$

n n $2n \times 2n$

$$\dot{x} = Ax + Bu$$
$$= Ax + B(-K\hat{x})$$
$$= Ax - BK\hat{x}$$
$$= Ax - BK(x - e)$$
$$= Ax - BKx + BKe$$
$$= (A-BK)x + BKe$$

Eigenvalues of $\begin{bmatrix} A-BK & BK \\ 0 & A-LC \end{bmatrix}$

are roots of $\begin{vmatrix} sI-A+BK & -BK \\ 0 & sI-A+LC \end{vmatrix} = 0$

or $\left| sI-(A-BK) \right| \left| sI-(A-LC) \right| = 0$

or $\underbrace{\mathrm{Eig}(A-BK)}_{\text{from Controller design}} \cup \underbrace{\mathrm{Eig}(A-LC)}_{\text{from Observer design}}$

Separation principle: Controller & Observer can be designed separately.

$$\begin{bmatrix} x \\ \hat{x} \end{bmatrix} = \underbrace{\begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}}_{2n \times 2n} \begin{bmatrix} x \\ e \end{bmatrix}$$

?

what is this transformation matrix

$$\begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}^{-1} = \begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}$$

$$\begin{bmatrix} x \\ e \end{bmatrix} = \begin{bmatrix} I & 0 \\ I & -I \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

as this is co-ordinate transformation, their matrices should be related

$$\begin{bmatrix} A & -BK \\ LC & A-BK-LC \end{bmatrix} T = T \begin{bmatrix} A-BK & BK \\ 0 & A-LC \end{bmatrix}$$

$T = ?$