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Kalman Filter . . .

1. Starting model needs to be updated.

$$\dot{x} = Ax + Bu + \underbrace{\text{process noise}}_{v}$$

$$y = Cx + \underbrace{\text{measurement noise}}_{w}$$

→ added noise

→ discrete time (mainly because it is easier)

$\boxed{\text{zero mean, "white" noise}}$

Kalman-Bucy filter for continuous time (AM Thm 8.5)

The optimal estimator has the form of a linear observer

$$\frac{d\hat{x}}{dt} = A\hat{x} + Bu + L(y - C\hat{x}), \quad \hat{x}(0) = E(x(0))$$

where $L = PC^T R_w^{-1}$ and

$$P = E\{(\overbrace{x(t) - \hat{x}(t)}^{e(t) = x - \hat{x}})(x(t) - \hat{x}(t))^T\}$$

$$\frac{dP}{dt} = AP + PA^T - PC^T R_w^{-1} CP + R_v$$

$$P(0) = E\{(x(0) - x_0)(x(0) - x_0)^T\}$$

$$e(t) = x(t) - \hat{x}(t)$$

We minimize $E\{e(t)\, e^T(t)\}$

Significance?

Possibly variance?

What is $E\{e(t)\} = E\{x\} - E\{\hat{x}\}$

$$\frac{d}{dt} E\{e(t)\} = \frac{d}{dt} E\{x\} - \frac{d}{dt} E\{\hat{x}\}$$

$$= E\left\{\frac{dx}{dt}\right\} - E\left\{\frac{d}{dt}\hat{x}\right\}$$

$$= E\left\{\frac{dx}{dt} - \frac{d\hat{x}}{dt}\right\}$$

$$= E\{Ax + \cancel{Bu} + v - A\hat{x} - \cancel{Bu} - L(y - C\hat{x})\}$$

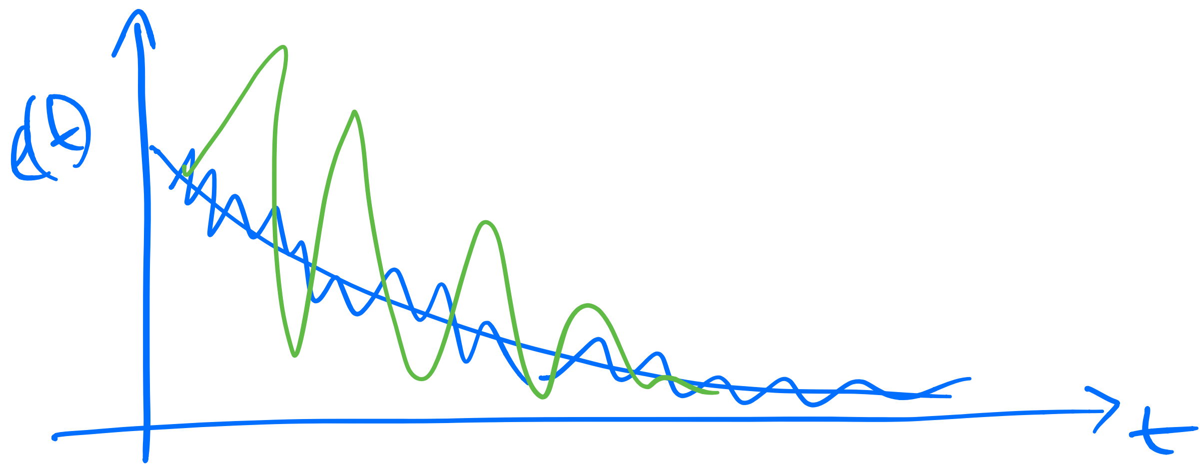
$$= E\{Ax + v - A\hat{x} - L(Cx + w - C\hat{x})\}$$

$$= E\{A(x - \hat{x}) - LC(x - \hat{x}) + v - Lw\}$$

$$= E\{(A - LC)e + v - Lw\}$$

$$= (A - LC)\, E\{e\} + E\{\cancelto{0}{v}\} - L E\{\cancelto{0}{w}\}$$

$$\frac{d}{dt} E\{e(t)\} = (A - LC)\, E\{e\}$$



Kalman Filter minimizes the error covariance at each time.

Discrete-time

$$x(k+1) = A x(k) + B u(k) + v(k)$$

$$y(k) = C x(k) + w(k)$$

v, w are zero-mean white noise

$\rightarrow$ Discrete-time Kalman Filter

$f(x) = x^2 - 2x + 1$

$\frac{df}{dx} = 2x - 2 = 0$

$x = 1$

$\frac{d^2f}{dx^2} = 2 > 0$

$(x-1)^2$

minimum is 0