ELL333

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noise co.variances

$$\therefore P_{k+1} = (A - L_k C) P_k (A - L_k C)' + R_v + L_k R_w L_k'$$

To minimize RHS by choosing L_k

(after some algebra)

$$= A P_k A' + R_v$$

$$+ (L_k - A P_k C' R_\epsilon^{-1}) R_\epsilon (L_k - A P_k C' R_\epsilon^{-1})'$$

$$- A P_k C' R_\epsilon^{-1} C P_k' A',$$

$$R_\epsilon = R_w + C P_k C'$$

matrix version

↳ minimizing all entries

↳ matrix positivity

Choose

$$L_k = A P_k C' R_\epsilon^{-1}$$

$$= A P_k C' (R_w + C P_k C')^{-1}$$

Check scalar case ✓

$$L_k = \frac{a c P_k}{R_w + c^2 P_k}$$

What does it mean

$$(L_k - A P_k C' R_\epsilon^{-1}) R_\epsilon (L_k - A P_k C' R_\epsilon^{-1})' \geq 0 ?$$

What does it mean minimize P_{k+1} at each time step?

matrix

$$P_k = E\{(x_k - \hat{x}_k)(x_k - \hat{x}_k)'\}$$

Check for $n=2$

$$E\left\{\begin{bmatrix} e_{1k} \\ e_{2k} \end{bmatrix}\begin{bmatrix} e_{1k} & e_{2k} \end{bmatrix}\right\}$$

$$= E\left\{\begin{bmatrix} e_{1k}^2 & e_{1k}e_{2k} \\ e_{2k}e_{1k} & e_{1k}^2 \end{bmatrix}\right\}$$

$$= \begin{bmatrix} E(e_{1k}^2) & E(e_{1k}e_{2k}) \\ E(e_{2k}e_{1k}) & E[e_{2k}^2) \end{bmatrix}$$

$[P_k]_{2\times 2}$ is symmetric

$\underline{n=2}$

$$\underset{2\times 2}{P_{k+1}} = (A - L_k C) P_k (A - L_k C)' + R_v + L_k R_w L_k'$$

↳ How many different scalar equations in this?

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

- $R_v = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$, R_w is scalar as scalar output

$$L_k = \begin{bmatrix} l_{1k} \\ l_{2k} \end{bmatrix}$$

- $$L_k R_w L_k' = \begin{bmatrix} l_{1k} \\ l_{2k} \end{bmatrix} R_w \begin{bmatrix} l_{1k} & l_{2k} \end{bmatrix}$$

$$= R_w \begin{bmatrix} l_{1k}^2 & l_{1k} l_{2k} \\ l_{1k} l_{2k} & l_{2k}^2 \end{bmatrix}$$

$A - L_k C$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} - \begin{bmatrix} l_{1k} & 0 \\ l_{2k} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} - l_{1k} & a_{12} \\ a_{21} - l_{2k} & a_{22} \end{bmatrix}$$

- $$(A - L_k C) P_k (A - L_k C)'$$

$$= \begin{bmatrix} a_{11} - l_{1k} & a_{12} \\ a_{21} - l_{2k} & a_{22} \end{bmatrix} \begin{bmatrix} E\{e_{1k}^2\} & E\{e_{1k} e_{2k}\} \\ E\{e_{1k} e_{2k}\} & E\{e_{2k}^2\} \end{bmatrix}$$

$$= \begin{bmatrix} (a_{11} - l_{1k}) E\{e_{1k}^2\} + a_{12} E\{e_{1k} e_{2k}\} & (a_{11} - l_{1k}) E\{e_{1k} e_{2k}\} + a_{12} E\{e_{2k}^2\} \\ (a_{21} - l_{2k}) E\{e_{1k}^2\} + a_{22} E\{e_{1k} e_{2k}\} & (a_{21} - l_{2k}) E\{e_{1k} e_{2k}\} + a_{22} E\{e_{2k}^2\} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} - l_{1k} & a_{21} - l_{2k} \\ a_{12} & a_{22} \end{bmatrix}$$

$$(1,1) = (a_{11} - l_{1k})^2 E\{e_{1k}^2\} + (a_{11} - l_{1k}) a_{12} E\{e_{1k} e_{2k}\}$$
$$+ a_{12} (a_{11} - l_{1k}) E\{e_{1k} e_{2k}\} + a_{12}^2 E\{e_{2k}^2\}$$

$$(1,2) = (a_{11} - l_{1k})(a_{21} - l_{2k}) E\{e_{1k}^2\} + a_{12}(a_{21} - l_{2k}) E\{e_{1k} e_{2k}\}$$

$$+ a_{22}(a_{11} - l_{1k}) E\{e_{1k} e_{2k}\} + a_{22} a_{12} E\{e_{2k}^2\}$$

$$(2,2) = (a_{21} - l_{2k})^2 E\{e_{1k}^2\} + 2a_{22}(a_{21} - l_{2k}) E\{e_{1k} e_{2k}\} + a_{22}^2 E\{e_{2k}^2\}$$

$$\Rightarrow E\{e_{1(k+1)}^2\} = (1,1) + v_{11} + R_w l_{1k}^2$$

$$E\{e_{1(k+1)} e_{2(k+1)}\} = (1,2) + v_{12} + R_w l_{1k} l_{2k}$$

$$E\{e_{2(k+1)}^2\} = (2,2) + v_{22} + R_w l_{2k}^2$$

$\longrightarrow$ better way?