ELL 333

31.10.2019

Kalman Filter

↳ Optimal Observer in presence of noise

<u>Way1</u> : Assume observer structure + minimize error covariance at each time

⟶ Kalman Gain

<u>Way2</u>: Minimize variance of state estimate ↳ like least squares

⟶ Kalman gain + Observer structure

- Probability
  - ↳ minimum variance estimate = conditional mean (minimum mean square estimate)
  - ↳ X & Y are jointly Gaussian, then

$$E(X|Y) = E(X) + \Sigma_{XY}\Sigma_Y^{-1}(Y - E(Y))$$

$$\Sigma_{X|Y} = \Sigma_X - \Sigma_{XY}\Sigma_Y^{-1}\Sigma_{XY}'$$

- Apply this to a discrete time linear system known

$$x_{k+1} = A x_k + v_k$$

$$y_k = C x_k + w_k$$

+ $x_0, \{v_k\}, \{w_k\}$ are independent, Gaussian

$x_0 \sim \mathcal{N}(\bar{x}_0, \Sigma_0)$

$v_k \sim \mathcal{N}(0, \Sigma_v)$

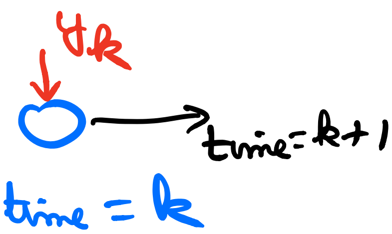
$w_k \sim \mathcal{N}(0, \Sigma_w)$

→ Measurement update

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + \Sigma_{k|k-1} C' \left(C \Sigma_{k|k-1} C' + \Sigma_w\right)^{-1} \left(y_k - C \hat{x}_{k|k-1}\right)$$

$$\Sigma_{k|k} = \Sigma_{k|k-1} - \Sigma_{k|k-1} C' \left(C \Sigma_{k|k-1} C' + \Sigma_w\right)^{-1} C \Sigma_{k|k-1}$$

→ Time update



$$\hat{x}_{k+1|k} = A \hat{x}_{k|k}$$

$$\Sigma_{k+1|k} = A \Sigma_{k|k} A' + \Sigma_v$$

Where is the observer hidden?

$$\hat{x}_{k+1|k} = A \hat{x}_{k|k-1} + \underbrace{A \Sigma_{k|k-1} C' \left(C \Sigma_{k|k-1} C' + \Sigma_w\right)^{-1}}_{\text{gain}} \left(y_k - C \hat{x}_{k|k-1}\right)$$

Quiz 11

$$z = Ax + v, \quad x \sim \mathcal{N}(\bar{x}, \Sigma_x)$$
$$v \sim \mathcal{N}(0, \Sigma_v)$$

x, v are independent

Find $E(z)$ &

$E\{(z-\bar{z})(z-\bar{z})'\}$

in terms of $\bar{x}$, Σ_x, Σ_v, and A.