ELL 333

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ELL 700 Minor Test 1 #3

have to calculate the generalized eigenvectors

ELL 700 Minor Test 2 # 3

$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad A = \begin{bmatrix} -4 & 5 \\ -1 & 2 \end{bmatrix}$$

$$[B \quad AB] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \rightarrow \text{rank} = 1$$

uncontrollable.

$$A+BF = \begin{bmatrix} -4 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} [f_1 \ f_2]$$

$$= \begin{bmatrix} -4 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} f_1 & f_2 \\ f_1 & f_2 \end{bmatrix}$$

$$= \begin{bmatrix} -4+f_1 & 5+f_2 \\ -1+f_1 & 2+f_2 \end{bmatrix}$$

$$\lambda I - (A+BF) = \begin{bmatrix} \lambda+4-f_1 & -5-f_2 \\ 1-f_1 & \lambda-2-f_2 \end{bmatrix}$$

$$= (\lambda+4-f_1)(\lambda-2-f_2) + (1-f_1)(5+f_2)$$

$$= \lambda^2 + (4 - f_1 \quad -2 - f_2)\lambda - (4 - f_1)(2 + f_2)$$
$$+ (1 - f_1)(5 + f_2)$$

$$= \lambda^2 + (2 - f_1 - f_2)\lambda$$
$$+ [5 - 5f_1 + f_2 - f_1 f_2$$
$$- 8 - 4f_2 + 2f_1 + f_1 f_2]$$

$$= \lambda^2 + (2 - f_1 - f_2)\lambda$$
$$+ (-3 - 3f_1 - 3f_2)$$

$$= \lambda^2 + (2 - f_1 - f_2)\lambda - 3(1 + f_1 + f_2)$$

roots -3

$1 + f_1 + f_2$

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prev question

$\dot{x} = Ax,$

A has $-1, -2$ → repeated two times

Jordan Canonical form, $J = \begin{bmatrix} e^{-t} & 0 & 0 \\ 0 & e^{-2t} & te^{-2t} \\ 0 & 0 & e^{-2t} \end{bmatrix}$

& $\exists$ invertible T such that $A = TJT^{-1}$

$T^{-1}AT = J$

General solution

$$x(t) = e^{At}\, x(0)$$

$$= T\, e^{Jt}\, \underbrace{T^{-1} \underbrace{x(0)}_{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}}}_{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}$$

$$e^{Jt} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} e^{-t} & 0 & 0 \\ 0 & e^{-2t} & te^{-2t} \\ 0 & 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$= \begin{bmatrix} d_1 e^{-t} \\ d_2 e^{-2t} + d_3 t e^{-2t} \\ d_3 e^{-2t} \end{bmatrix}$$

$$= e^{-t} \begin{bmatrix} d_1 \\ 0 \\ 0 \end{bmatrix} + e^{-2t} \begin{bmatrix} 0 \\ d_2 \\ d_3 \end{bmatrix} + t e^{-2t} \begin{bmatrix} 0 \\ d_3 \\ 0 \end{bmatrix}$$

$$x(t) = e^{-t}\, T \begin{bmatrix} d_1 \\ 0 \\ 0 \end{bmatrix} + e^{-2t}\, T \begin{bmatrix} 0 \\ d_2 \\ d_3 \end{bmatrix} + t e^{-2t}\, T \begin{bmatrix} 0 \\ d_3 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -8 & -5 \end{bmatrix}$$

$$x(t) = e^{At} x(0)$$

$$\quad\quad\quad\quad\quad \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

$$A^3 + 5A^2 + 8A + 4I = 0$$

$$\Rightarrow A^3 = -5A^2 - 8A - 4I$$

$$e^{At} = I + tA + \frac{t^2}{2!} A^2 + \frac{t^3}{3!} A^3 + \frac{t^4}{4!} A^4 + \cdots$$

$$A^3 = -5A^2 - 8A - 4I$$

$$A^4 \ \cdots$$

$$A^n = d_n A^2 + e_n A + f_n I$$

$$\therefore \quad e^{At} = g_1(t) I + g_2(t) A + g_3(t) A^2$$

$$\therefore x(t) = e^{At} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

$$= g_1(t) \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} + g_2(t) \begin{bmatrix} c_2 \\ c_3 \\ -4c_1 - 8c_2 - 5c_3 \end{bmatrix} + g_3(t) \begin{bmatrix} c_3 \\ -4c_1 - 8c_2 - 5c_3 \\ -4c_2 - 8c_3 + 20c_1 + 40c_2 + 25c_3 \end{bmatrix}$$

$$= g_1(t) \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} + g_2(t) \begin{bmatrix} c_2 \\ c_3 \\ -4c_1 - 8c_2 - 5c_3 \end{bmatrix}$$

$$+ g_3(t) \begin{bmatrix} c_3 \\ -4c_1 - 8c_2 - 5c_3 \\ 20c_1 + 36c_2 + 17c_3 \end{bmatrix}$$