

ELL 333
08.11.2019

Major Test
LH108
1-3PM
19.11.2019

Kalman Filter Convergence

"Cobweb" for one dimensional maps

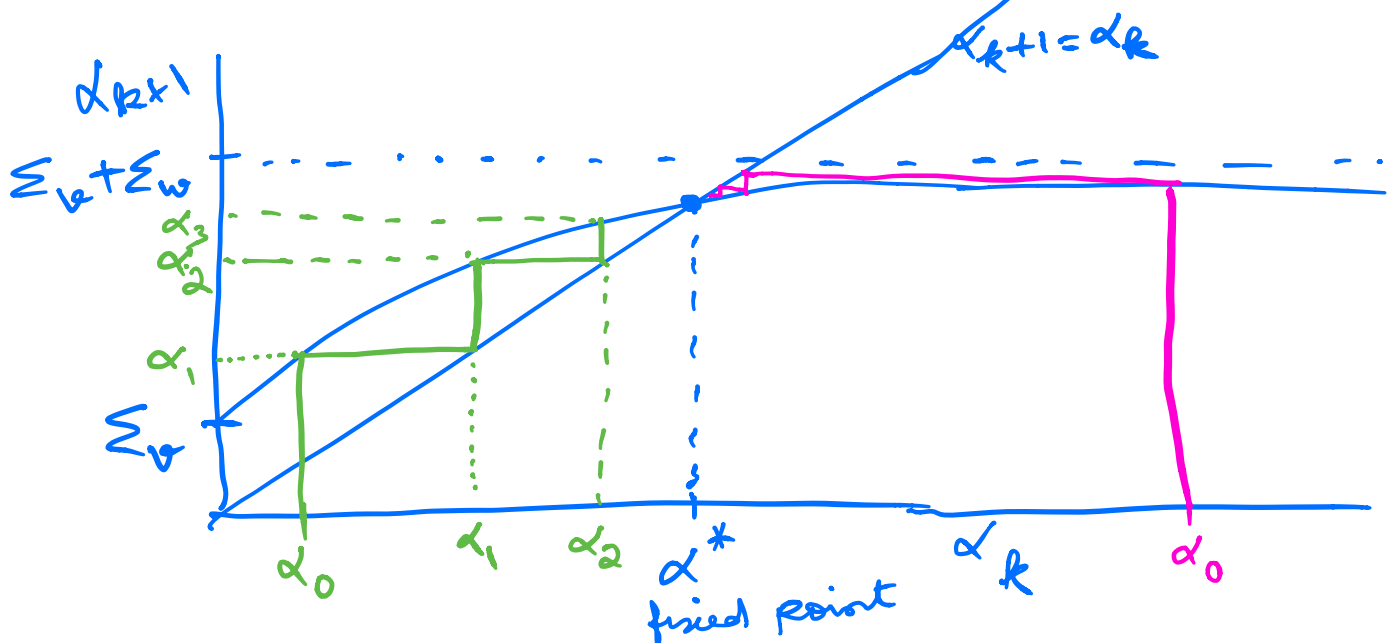
$$\alpha_{k+1} = \frac{4 \sum_{k|k-1} \Sigma_w}{\sum_{k|k-1} + \Sigma_w} + \Sigma_v$$

Diagram showing the derivation of the Kalman filter update equation. The left side is α_{k+1} with a downward arrow. The right side is a fraction with $4 \sum_{k|k-1} \Sigma_w$ in the numerator and $\sum_{k|k-1} + \Sigma_w$ in the denominator, plus Σ_v . Arrows point from the terms in the fraction to the corresponding terms in the equation above.

$$\Rightarrow \alpha_{k+1} = \frac{4 \alpha_k \Sigma_w}{\alpha_k + \Sigma_w} + \Sigma_v$$

Map: $\alpha_k \xrightarrow{f} \alpha_{k+1}$

Fixed points: $f(\alpha_k) = \alpha_k$



For global stability of one-dimensional maps,
draw above cobweb like structure.

For local behaviour around $\alpha = \alpha^*$,
expand in a Taylor Series,

$$\alpha_{k+1} = f(\alpha_k)$$

Expand $f(\alpha_k)$ around $\alpha_k = \alpha^*$

$$\Rightarrow \alpha_{k+1} = f(\alpha_k)$$

$$= f(\alpha^*) + \left. \frac{df}{d\alpha_k} \right|_{\alpha^*} (\alpha_k - \alpha^*) + \frac{1}{2} \left. \frac{d^2 f}{d\alpha_k^2} \right|_{\alpha^*} (\alpha_k - \alpha^*)^2 + \dots$$

$$= \alpha^* + \left. \frac{df}{d\alpha_k} \right|_{\alpha^*} (\alpha_k - \alpha^*) + \frac{1}{2} \left. \frac{d^2 f}{d\alpha_k^2} \right|_{\alpha^*} (\alpha_k - \alpha^*)^2 + \dots$$

$$\alpha_{k+1} - \alpha^* = \left. \frac{df}{d\alpha_k} \right|_{\alpha^*} (\alpha_k - \alpha^*) + \frac{1}{2} \left. \frac{d^2 f}{d\alpha_k^2} \right|_{\alpha^*} (\alpha_k - \alpha^*)^2 + \dots$$

$\underbrace{\hspace{10em}}$
 $\Delta \alpha_{k+1}$ or

$$\beta_{k+1} = \left. \frac{df}{d\alpha_k} \right|_{\alpha^*} \beta_k + (\quad) \beta_k^2 + \dots$$

if very close to α^*

$$\beta_{k+1} \approx \left. \frac{df}{d\alpha_k} \right|_{\alpha^*} \beta_k$$

Calculate $\left. \frac{df}{d\alpha_k} \right|_{\alpha^*}$. If $< 1 \Rightarrow$ Stable
If $> 1 \Rightarrow$ Unstable

Local Stability

\Downarrow If $= 1$, cannot conclude anything for nonlinear map, consider higher order terms.

Quiz 12

Q. Consider the one dimensional map,

$$x_{k+1} = x_k^2.$$

Find the fixed point(s) and analyse their stability by drawing "cobwebs".