ELL 333
08.11.2019

Major Test
LH108
1-3PM
19.11.2019

Kalman Filter Convergence

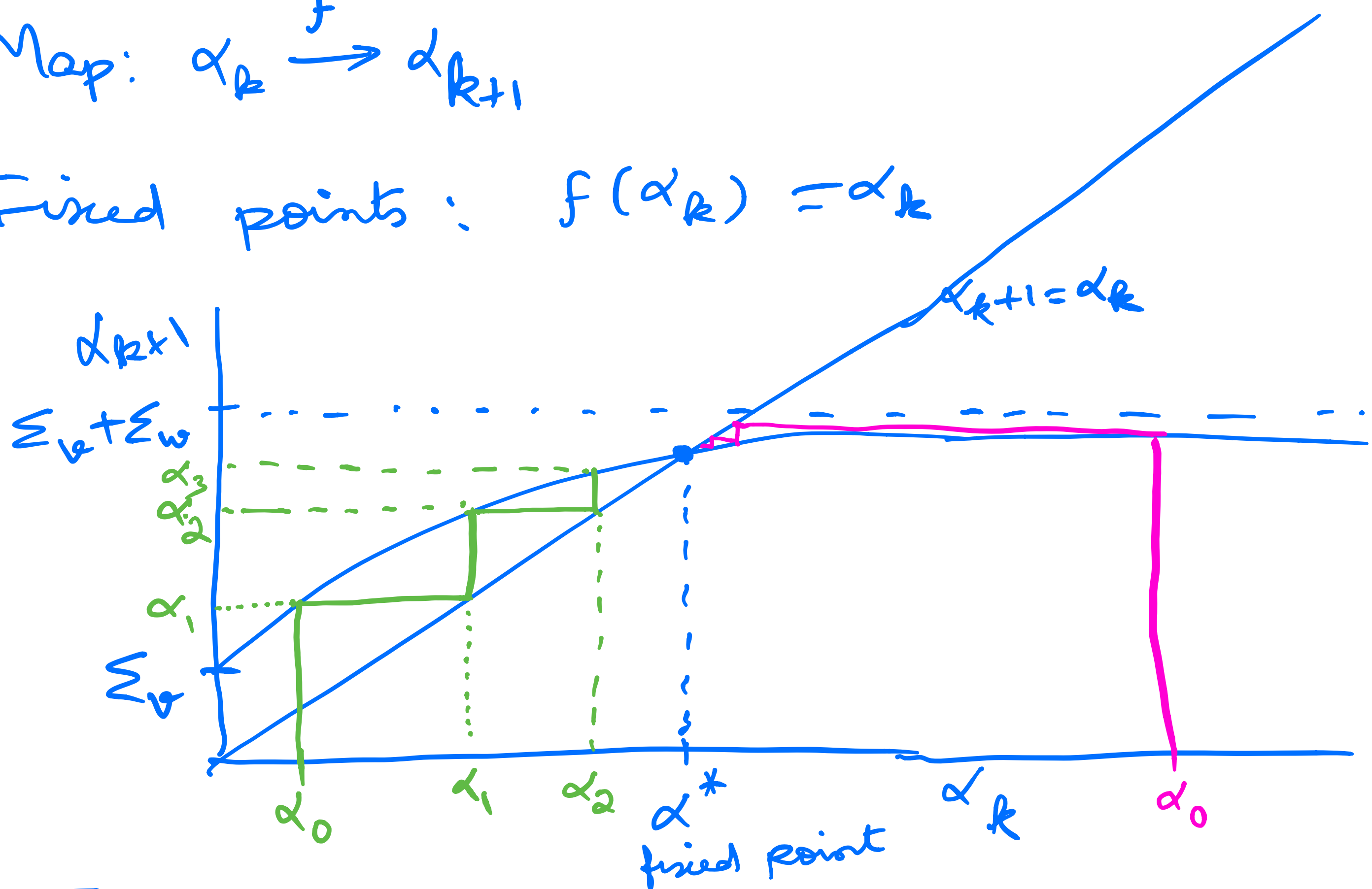
"Cobweb" for one dimensional maps

$$\underbrace{\Sigma_{k+1|k}}_{\alpha_{k+1}} = \frac{4\,\Sigma_{k|k-1}\Sigma_w}{\underbrace{\Sigma_{k|k-1}}_{\alpha_k} + \Sigma_w} + \Sigma_v$$

$$\Rightarrow \quad \alpha_{k+1} = \frac{4\,\alpha_k \Sigma_w}{\alpha_k + \Sigma_w} + \Sigma_v$$

Map: $\alpha_k \xrightarrow{f} \alpha_{k+1}$

Fixed points: $f(\alpha_k) = \alpha_k$



For global stability of one-dimensional maps draw above cobweb like structure.

For local behaviour around $\alpha = \alpha^*$, expand in a Taylor Series,

$$\alpha_{k+1} = f(\alpha_k)$$

Expand $f(\alpha_k)$ around $\alpha_k = \alpha^*$

$$\Rightarrow \alpha_{k+1} = f(\alpha_k)$$

$$= f(\alpha^*) + \left.\frac{df}{d\alpha_k}\right|_{\alpha^*} (\alpha_k - \alpha^*) + \frac{1}{2} \left.\frac{d^2 f}{d\alpha_k^2}\right|_{\alpha^*} (\alpha_k - \alpha^*)^2 + \ldots$$

$$= \alpha^* + \left.\frac{df}{d\alpha_k}\right|_{\alpha^*} (\alpha_k - \alpha^*) + \frac{1}{2} \left.\frac{d^2 f}{d\alpha_k^2}\right|_{\alpha^*} (\alpha_k - \alpha^*)^2 + \ldots$$

$$\underbrace{\alpha_{k+1} - \alpha^*}_{\Delta\alpha_{k+1} \text{ or } \beta_{k+1}} = \left.\frac{df}{d\alpha_k}\right|_{\alpha^*} (\alpha_k - \alpha^*) + \frac{1}{2} \left.\frac{d^2 f}{d\alpha_k^2}\right|_{\alpha^*} (\alpha_k - \alpha^*)^2 + \ldots$$

$$\beta_{k+1} = \left.\frac{df}{d\alpha_k}\right|_{\alpha^*} \beta_k + (\ ) \beta_k^2 + \ldots$$

if very close to α^*

$$\beta_{k+1} \simeq \left.\frac{df}{d\alpha_k}\right|_{\alpha^*} \cdot \beta_k$$

Local Stability

Calculate $\left.\frac{df}{d\alpha_k}\right|_{\alpha^*}$. If $< 1 \Longrightarrow$ Stable

If $> 1 \Longrightarrow$ Unstable

If $= 1$, cannot conclude anything for nonlinear map, consider higher order terms.

Quiz 12

Q. Consider the one dimensional map,

$$x_{k+1} = x_k^2 .$$

Find the fixed point(s) and analyze their stability by drawing "cobwebs".