ELL333

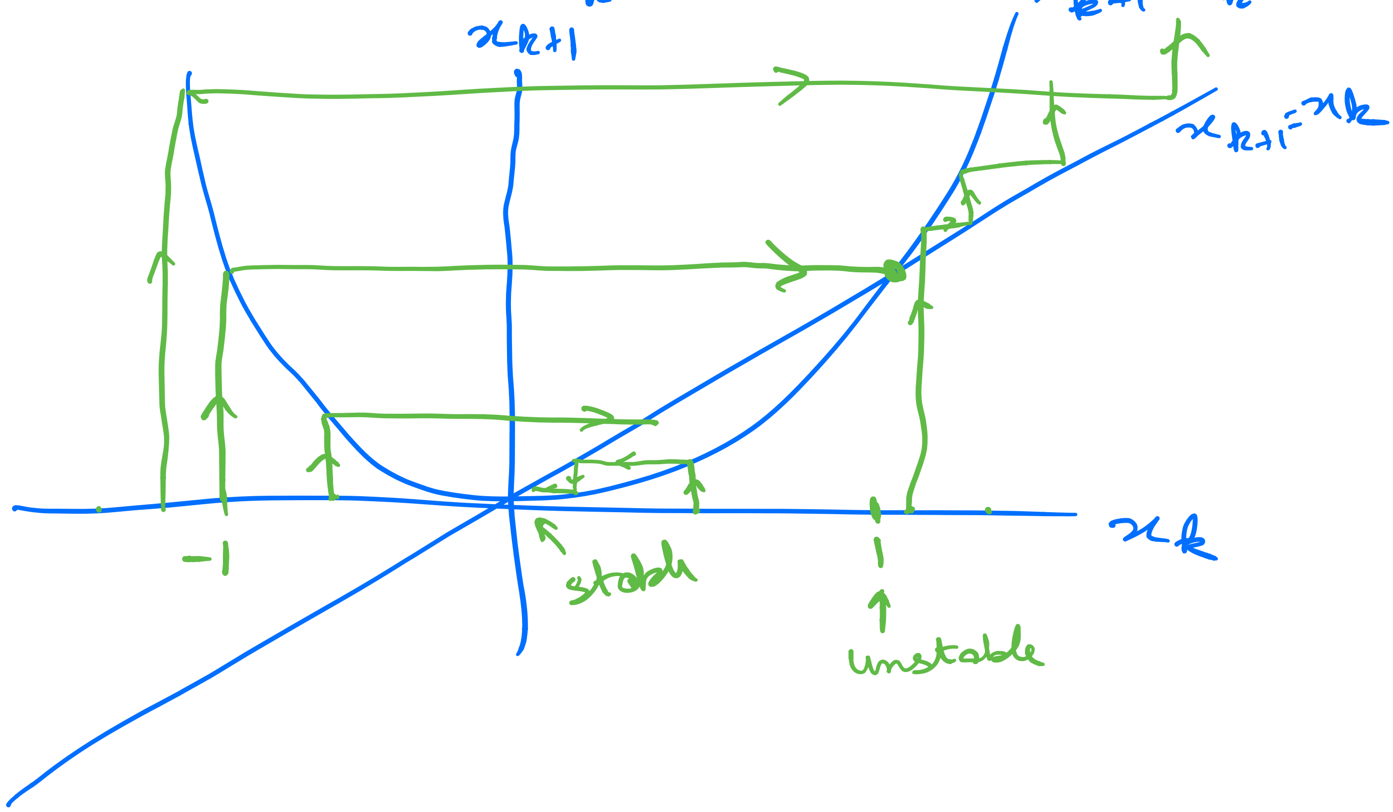
14.11.2019

Quiz 12. $x_{k+1} = x_k^2$

fixed point $x_{k+1} = x_k$

or $x_k = x_k^2$

$\Rightarrow x_k = 0, 1$



ELL700

M II. 3

Design $u = \underset{\to -K}{F} x$ s.t all closed loop eigenvalues satisfy $Re(\lambda) \le -2$

$$\dot{x} = \underbrace{\begin{bmatrix} -4 & 5 \\ -1 & 2 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{B} u$$

eig(A)?

$\lambda^2 + 2\lambda - 3 = 0$

$(\lambda + 3)(\lambda - 1)$

- $W_c = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

  $\text{rank}(W_c) = 1$

- linearly independent column vector

  $f = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

- What is another vector which when added to $\{f\}$ makes a basis for $\mathbb{R}^2$? (plane)

  $v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \ldots$

  Basis is $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$ ✓

- $T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} f & v \end{bmatrix}$

  $AT = \begin{bmatrix} Af & Av \end{bmatrix}$

  $Af = \begin{bmatrix} -4 & 5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1.f + 0.v$

  $Av = \begin{bmatrix} -4 & 5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -9 \\ -3 \end{bmatrix} = \underset{-6}{\alpha} f + \underset{-3}{\beta} v$

  $\alpha + \beta = -9$

  $\alpha - \beta = -3$

$$AT = [Af \;\; Av] = [1.f + 0.v \quad -6.f - 3.v]$$

$$= \underbrace{[f \;\; v]}_{T} \begin{bmatrix} 1 & -6 \\ 0 & -3 \end{bmatrix}$$

$$\Rightarrow \underbrace{T^{-1} A T}_{\triangleq \hat{A}} = \begin{bmatrix} 1 & -6 \\ 0 & -3 \end{bmatrix}$$

This is a co-ordinate transformation

$$\dot{x} = Ax + Bu \qquad x = T\hat{x}$$

$$\longrightarrow \dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u \qquad \hat{A} = T^{-1}AT$$

$$\hat{B} = T^{-1}B$$

As B is a column of W,

$B = 1.f + 0.v$ is not surprising

$$= \underbrace{[f \;\; v]}_{T} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$T^{-1}B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \hat{B}$$

- $$\dot{\hat{x}} = \left[\begin{array}{c|c} 1 & -6 \\ \hline 0 & -3 \end{array}\right] \hat{x} + \left[\begin{array}{c} 1 \\ \hline 0 \end{array}\right] u$$

$$\dot{\hat{x}}_1 = \hat{x}_1 - 6\hat{x}_2 + u$$

$$\dot{\hat{x}}_2 = -3\hat{x}_2$$

We have split the space into controllable and uncontrollable subspaces

- In general,

$$\dot{x} = Ax + Bu$$

suppose $W_c = [B \;\; AB \; \ldots \; A^{n-1}B]$

and $\text{rank}(W_c) = r < n$

① Choose LI columns $\{f_1, f_2 \ldots f_r\}$

② Add $n-r$ LI vectors $\{v_1, v_2 \ldots v_{n-r}\}$ to make basis

Then there exists an invertible transformation T such that

$$T^{-1}AT = \tilde{A} = \begin{array}{cc} \begin{array}{cc} \overset{r}{\longleftrightarrow} & \overset{n-r}{\longleftrightarrow} \end{array} & \\ \left[\begin{array}{c|c} \tilde{A}_{11} & \tilde{A}_{12} \\ \hline 0 & \tilde{A}_{22} \end{array}\right] & \begin{array}{l} \updownarrow r \\ \updownarrow n-r \end{array} \end{array}$$

$$T = [f_1, f_2 \ldots f_r, v_1, v_2 \ldots v_{n-r}]$$

$$T^{-1}B = \tilde{B} = \left[\begin{array}{c} \tilde{B}_1 \\ \hline 0 \end{array}\right] \begin{array}{l} \updownarrow r \\ \updownarrow n-r \end{array}$$

and $(\tilde{A}_{11}, \tilde{B}_1)$ is controllable

(i.e. $\text{rank}\{[\tilde{B}_1 \;\; \tilde{A}_{11}\tilde{B}_1 \; \ldots \; \tilde{A}_{11}^{r-1}\tilde{B}_1]\} = r$