ELL 333 REMINOR TEST DURATION = 1 HR, MARKS = 20

1. Consider the system $\dot{x} = x + u$, $y = x$.

a) Is it stable?

b) Design a controller $u = -kx$ so that the eigenvalue is at $\lambda_c < 0$.

c) Design an observer with eigenvalue $\lambda_o < 0$ to obtain state estimate $\hat{x}$. (This might seem superfluous as the only state is measured. Design observer anyway in case there is noise etc. and for part d) below.)

d) When a state estimate dependent control law $u = -k\hat{x}$, k is from part a), is used explicitly calculate the time evolution of state $x(t)$ and the estimation error $e(t) = x(t) - \hat{x}(t)$ from initial conditions $x(0)$ and $e(0)$.

e) Sketch $x(t)$ vs t for $|\lambda_c| >> |\lambda_o|$, $|\lambda_o| << |\lambda_c|$, and $|\lambda_c| = |\lambda_o|$.

f) Based on part e) above, how would you choose $|\lambda_o| / |\lambda_c|$? Justify clearly.

$1 + 1 + 1 + 2 + 2 + 1 = 8$ marks

2. Suppose that we wish to estimate the position of a particle that is undergoing

a random walk in one dimension (i.e., along a line). We model the position of the particle as $x_{k+1} = x_k + u_k$, where x is the position of the particle and u is a white noise process with $E\{u_i\} = 0$ and $E\{u_i u_j\} = R_u \delta(i-j)$. We assume that we can measure x subject to additive, zero-mean, Gaussian white noise with covariance 1.

a) Compute the expected value and covariance of the particle as a function of k.

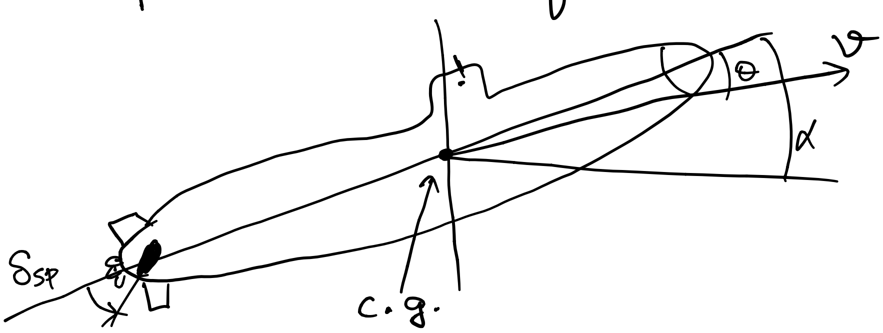
b) Construct a Kalman filter to estimate the position of the particle given the noisy measurements of its position.

c) Show that the filter converges.

d) Compute the steady-state expected value and covariance of the error of estimate.

$2 + 2 + 2 + 2 = 8$ marks

3. $A = \begin{bmatrix} 0 & 1 & 0 \\ -0.008 & -0.15 & 0.12 \\ 0 & 0.06 & -0.4 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ -0.1 \\ 0.8 \end{bmatrix}$ for depth control of a submarine. Is it controllable?



4 marks