ELL707

06.01.2020

solution structure
ex: limit cycle
steady-states

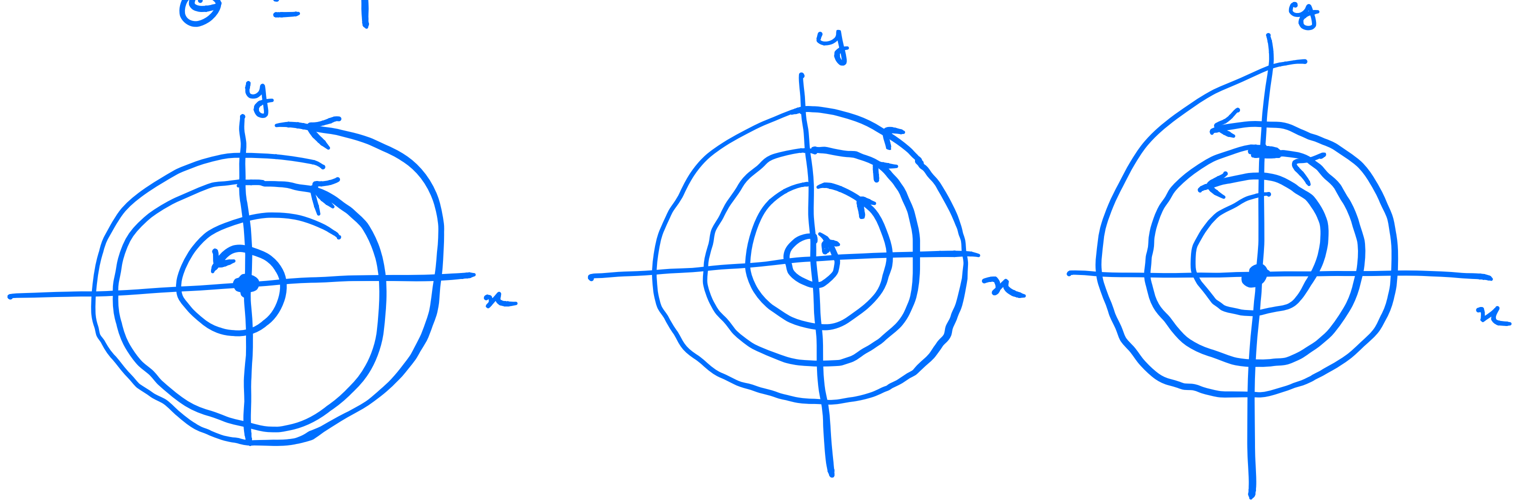
"Bifurcation"

As a parameter changes, the dynamics of the system change qualitatively.

↳ stability of fixed points / limit cycle changes.

$$\dot{r} = \mu r (1 - r^2)$$

$$\dot{\theta} = 1$$

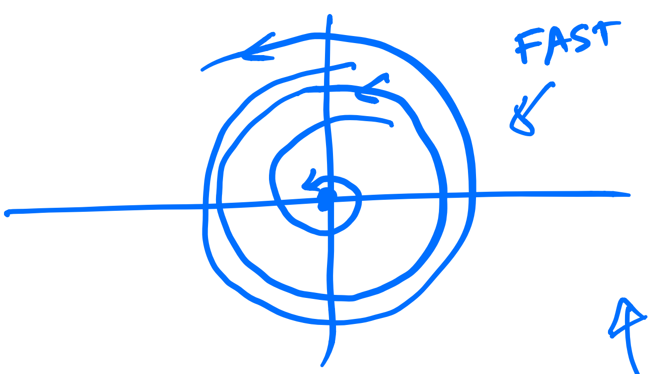


$\mu < 0$ $\qquad$ $\mu = 0$ $\qquad$ $\mu > 0$

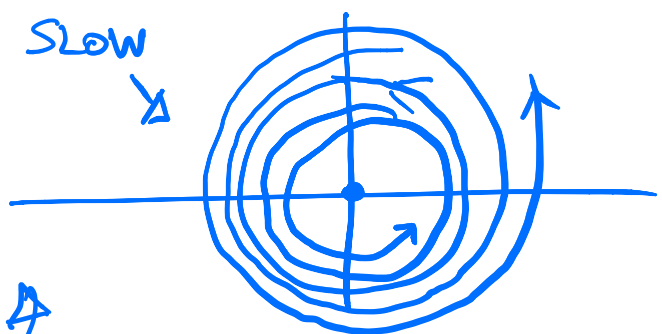
Bifurcation point

Bifurcation is Not a quantitative change

FAST



SLOW



$\mu = -10^{6}$ $\qquad$ $\mu = -10^{-6}$

Example of a quantitative change.

Oscillations

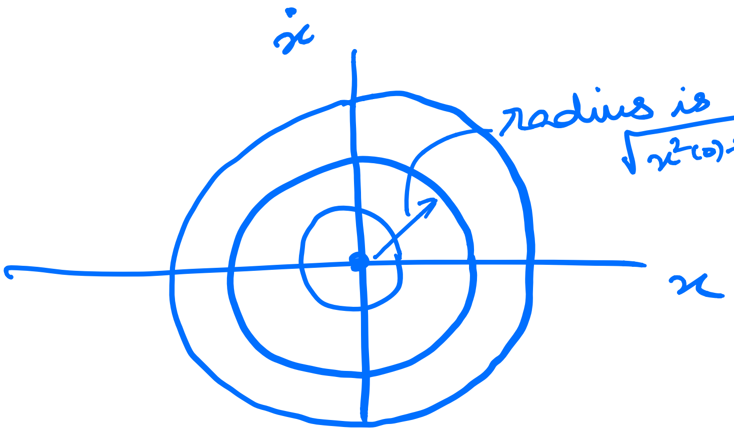
Example 4: Limit cycle with x and y as variables, (not r & θ).

4a) $\ddot{x} + x = 0$, $x(0)$, $\dot{x}(0)$

$\ddot{x} \to \frac{d^2x}{dt^2}$

$\boxed{\text{SOLVE}}$ $\to$ $x(t) = A\cos t + B\sin t$

$A \leftarrow x(0)$, $B \leftarrow \dot{x}(0)$



$x(t) = x(0)\cos t + \dot{x}(0)\sin t$

$\dot{x}(t) = -x(0)\sin t + \dot{x}(0)\cos t$

$x^2(t) + \dot{x}^2(t)$

$= x^2(0) + \dot{x}^2(0)$

Same solutions as $\dot{r} = 0$, $\dot{\theta} = 1$.

What could be transformation between these systems?

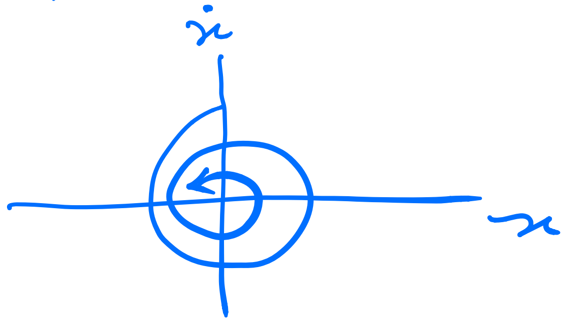
Question: Systems on a Line vs Systems on a Plane

What is a state? What is dimension?

$-\infty < x < \infty$

$0 < \theta \leq 2\pi$

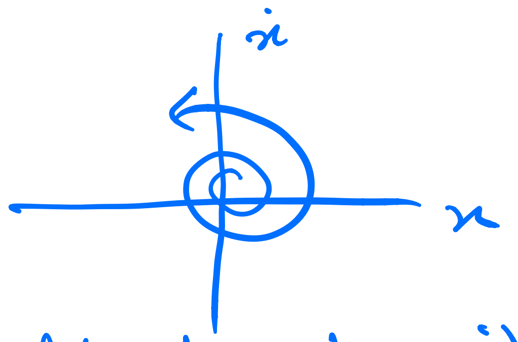
4b) $\ddot{x} + b\dot{x} + x = 0$, $b > 0$



How does this change?

- damped sinusiods $\to$ spiral

If $b<0$



What should $b = b(x, \dot{x})$ be for a limit cycle to exist?