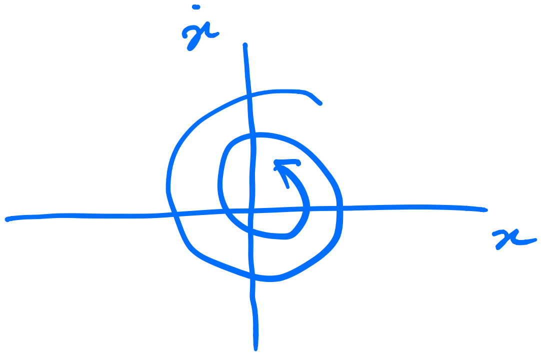
08.01.2020

ELL707

Damped Harmonic Oscillator

$$\ddot{x} + b\dot{x} + x = 0 \quad , b>0$$



What should $b = b(x,\dot{x})$ be for there to be a limit cycle?

MATLAB/pplane8 to simulate differential equations

$$\dot{x} = y$$
$$\dot{y} = -x - b\,y \rightarrow x, y, \ldots$$

$\hookrightarrow +1, +0.1$

$\hookrightarrow 0$

$\hookrightarrow -0.1 / -0.01$

Van der Pol oscillator

?

$$\ddot{x} + \underbrace{b(1-x^2)}_{b(x,\dot{x})}\dot{x} + x = 0 \quad , b>0$$

$\hookrightarrow -$

Spiralling in and Spiralling out
(at large distance from origin) (near the origin)

are the characteristic geometric behaviour of limit cycles as viewed from the polar co-ordinate examples

Here, same is replicated through the $(1-x^2)$ term.

$$|x| >> 1 \implies -b(1-x^2) > 0$$ POSITIVE DAMPING

$$|x| << 1 \implies -b(1-x^2) < 0$$ NEGATIVE DAMPING