ELL707

12.02.2020

Denominator polynomial with delay:

$$D(s) \equiv (s+1) + K e^{-Ts} = 0$$

Does it have purely imaginary roots?

Set $s = j\omega$

$\Rightarrow \quad j\omega + 1 = -K e^{-Tj\omega}$

$\Rightarrow \quad \sqrt{1+\omega^2} = |K|$

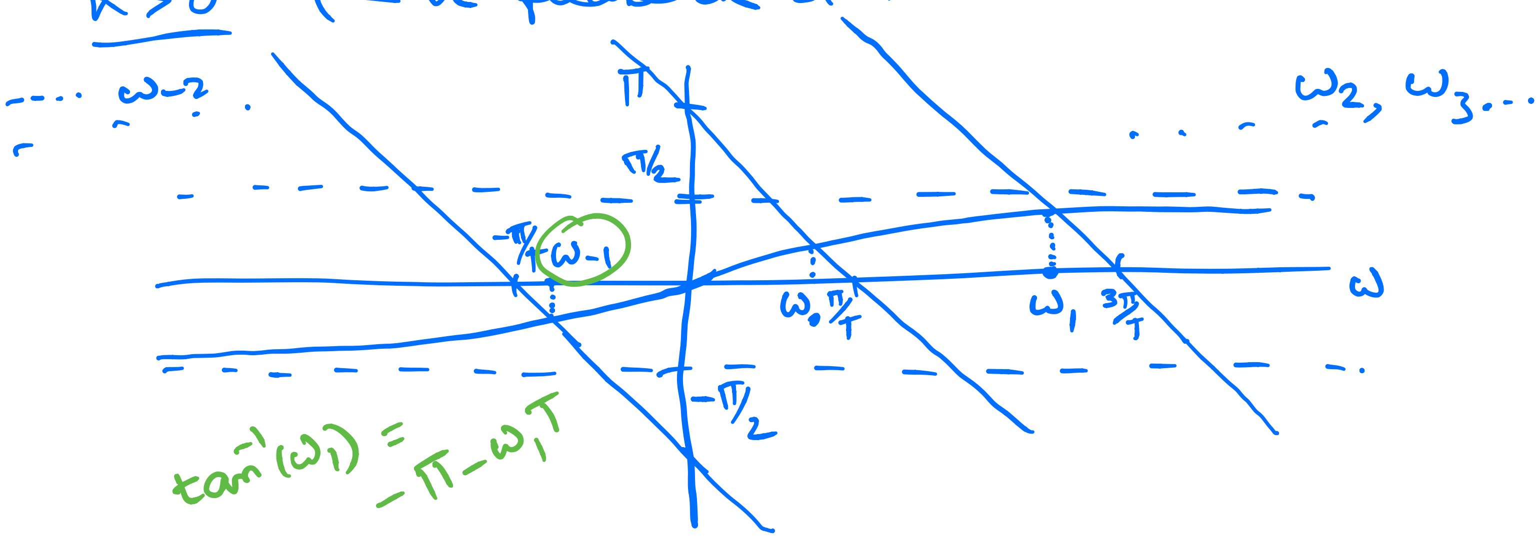
$\tan^{-1}\omega = \pi - \omega T + 2n\pi, \; n = 0, \pm 1, \pm 2$

if $K>0$

otherwise, π is not there

$\left\{ \begin{array}{l} K \neq 0 \\ K > 0 \\ K < 0 \end{array} \right.$ lets check when purely imaginary roots of $D(s)=0$ exist

$K>0$ (−ve feedback at $s=0$)



at $\omega = \omega_0$, $(\neq 0)$, there is an ~~pair of~~ imaginary root~~s~~ for $K = \sqrt{1+\omega_0^2}$

look up: N. Nise
K. Ogata
Dorf & Bishop } section on root locus with delay

$\therefore$ at $\omega = \omega_0, \omega_{\pm 1}, \omega_{\pm 2} \ldots$ there will be an ~~pair of~~ purely imaginary root~~s~~ with $K = \sqrt{1+\omega^2}$

Suppose we set $s = -j\omega$ $\quad (K > 0)$

$$1 - j\omega = -K e^{j\omega T}$$

$$\Rightarrow -\tan^{-1}(\omega) = \pi + \omega T + 2n\pi, \quad n = 0, \pm 1, \ldots$$

$\underline{\underline{n=0}}$

$$-\tan^{-1}(\omega) = \pi + \omega T$$

$$\Rightarrow \tan^{-1}(\omega) = \underline{\underline{-\pi}} - \omega T$$

point: $n=0$ equation for $s = -j\omega$ is the same as $n=-1$ equation for $s = +j\omega$.

$\therefore$ Pair of imaginary roots: $\{ \pm j\omega_0, \pm j\omega_1, \ldots \}$ (check!)

for $K = \{ \sqrt{1+\omega_0^2}, \sqrt{1+\omega_1^2}, \ldots \}$

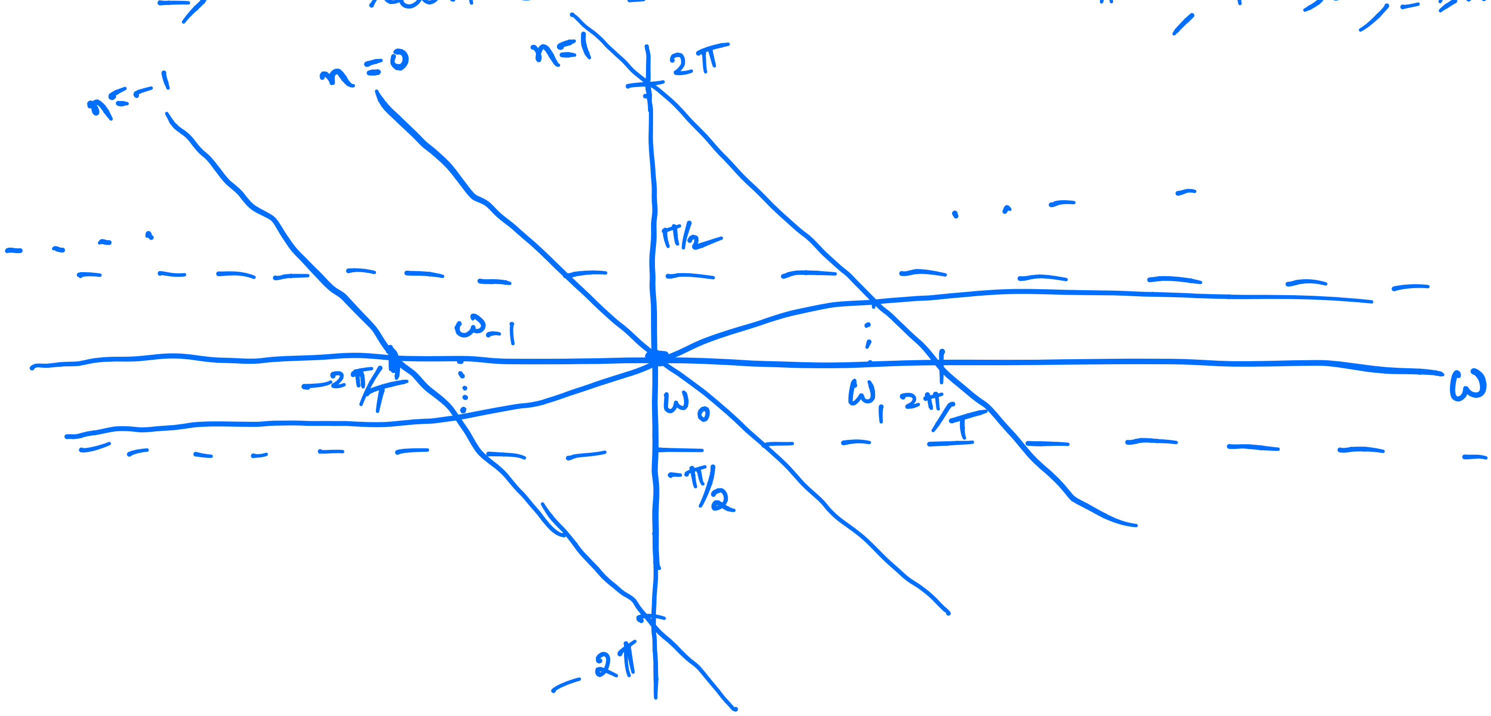
What happens for $K < 0$?

$$s + 1 = -K\, e^{-Ts}$$

$s = j\omega$, $\quad -K > 0$

$$\Rightarrow \quad j\omega + 1 = (-K)\, e^{-j\omega T}$$

$$\Rightarrow \quad \tan^{-1}\omega = -\omega T + 2n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$$



For $n = 0$, no purely imaginary pair of roots. just $\omega_0 = 0$, $K = -1$

For $n = 1$, purely imaginary pair of roots may exist $\pm j\omega_1$, $K = -\sqrt{1 + \omega_1^2}$

→ This makes sense that we don't expect oscillatory behaviour.

→ What does this mean?

experimental examples

design principles for oscillations??

- feedback (negative)
- delay

theoretical analysis

- examples
- linearized analysis (imaginary roots)

↳ Poincaré-Bendixson Theorem, ...

simulations

- with delay

as exact solutions not available

↳ describing function