Quiz 4

1. Consider the equation

$$s + r + K e^{-Ts} = 0,$$

s is complex number

$T > 0$ is real number

K is real number

$r > 0$ is real number

Does this have a pair of purely imaginary roots?

ELL707

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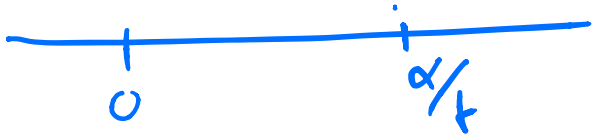
$n = 1$

$$\dot{x}_1 = f(x_1) - r x_1$$

Claim: $M = [0, \frac{\alpha}{r}]$ is positively invariant.

[if $x(0) \in M \Rightarrow x(t) \in M$ $\forall t \geq 0$

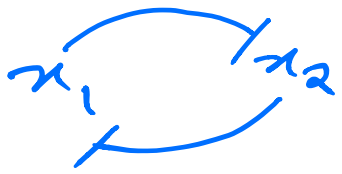


$x_1 = 0 \Rightarrow \dot{x}_1 = f(0) - 0 = \alpha > 0$

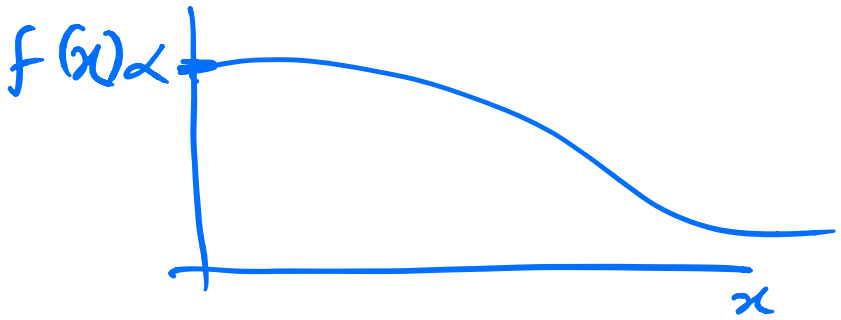
$x_1 = \frac{\alpha}{r} \Rightarrow \dot{x}_1 = f(\frac{\alpha}{r}) - \alpha < 0$

$n = 2$

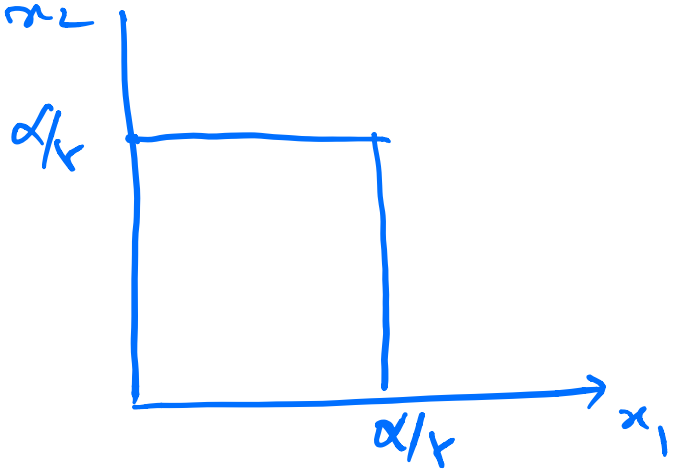
$$\dot{x}_1 = f(x_2) - x_1$$
$$\dot{x}_2 = f(x_1) - x_2$$



Sec 5.3 of DM



Claim: $M = [0, \frac{\alpha}{2}] \times [0, \frac{\alpha}{2}]$ is trvly invariant



$x_1 = 0$

$$\dot{x}_1 = f(x_2) > 0$$
$$\dot{x}_2 = f(0) - x_2 \quad (f(0) = \alpha)$$

Try for other edges of the square.