ELL 707

24.02.2020

$$(s+r)^3 - \underbrace{K_1 K_2 K_3}_{<0} = 0$$

Roots of this give stability of fixed point. If unstable (negative real part), then limit cycle exists.

What are the roots of this?

$$(s+r)^3 + K = 0, \quad K > 0 \quad [\because \text{ as } K_1, K_2, K_3 \text{ are all negative}$$

one way

$$(s+r)^3 = K(-1)$$

$$s = -r + K^{1/3}(-1)^{1/3}$$

$$-1$$

$$+\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

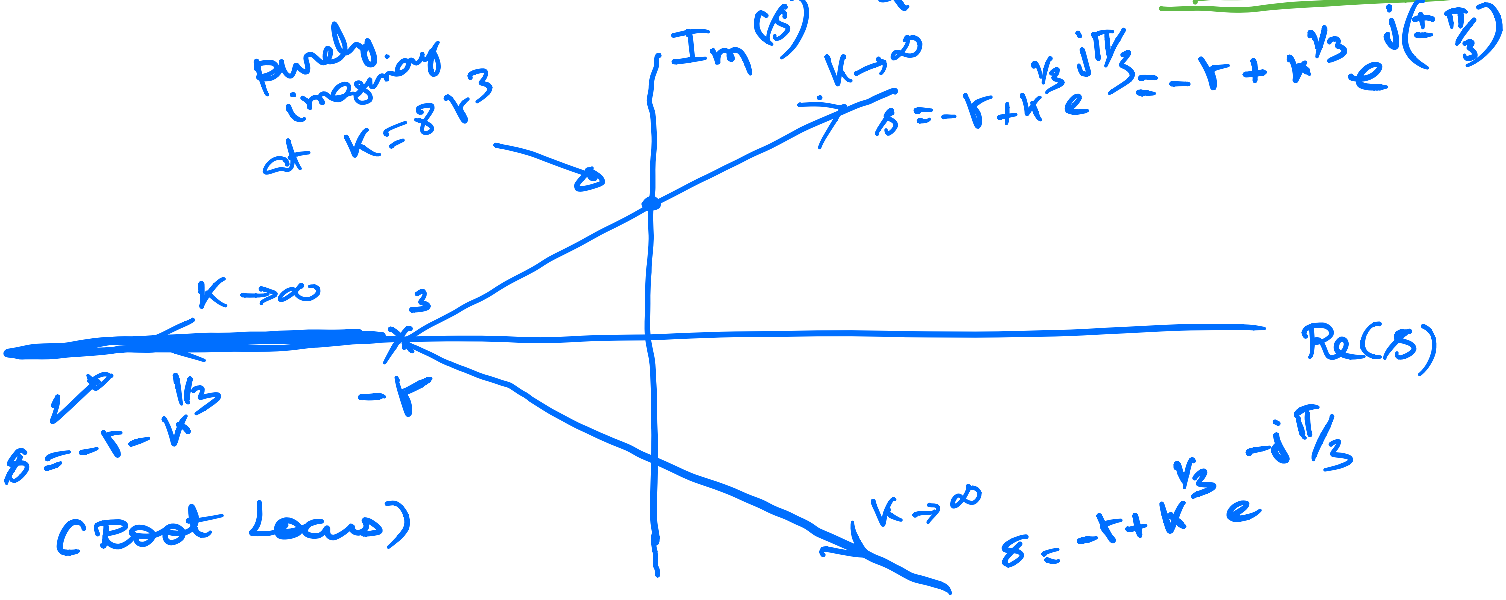
e^{j1}

What are cube roots of unity?

$$e^{j\pi} = -1$$

$$(-1)^{1/3} = e^{j\pi/3},\ e^{j\pi},\ e^{j5\pi/3}$$

$$s = -r + \frac{K^{1/3}}{2} \pm j\frac{K^{1/3}\sqrt{3}}{2}$$



another <u>way</u>

$s = \sigma + j\omega$, σ is real part

ω is imaginary part

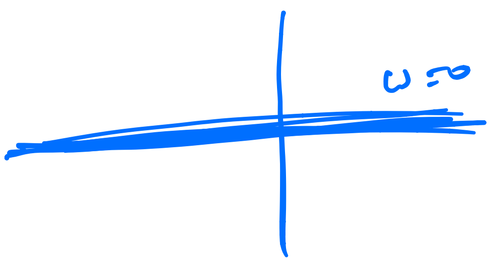
$$(\sigma + j\omega + r)^3 + K = 0$$

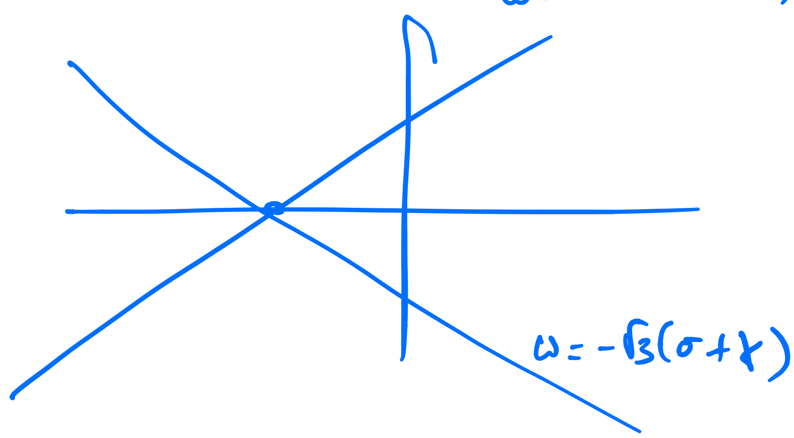
$$(\sigma + r)^3 + 3j\omega(\sigma + r)^2 + 3(\sigma + r)(j\omega)^2 + (j\omega)^3 + K = 0$$

$$\Rightarrow (\sigma + r)^3 - 3(\sigma + r)\omega^2 + \underbrace{K}_{>0} = 0$$

and $3\omega(\sigma + r)^2 - \omega^3 = 0$

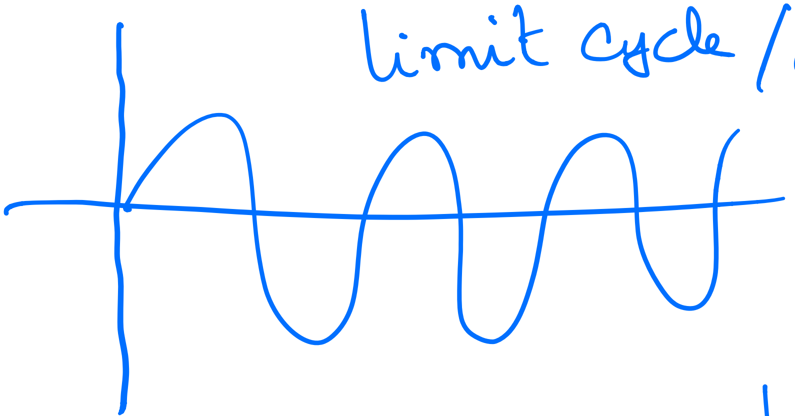
$\hookrightarrow$ $\omega = 0$, $\omega^2 = 3(\sigma + r)^2$

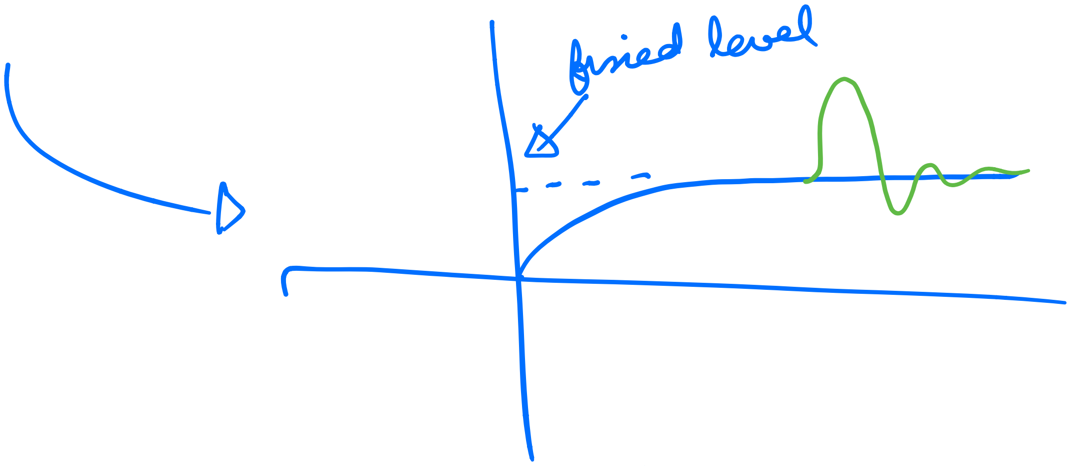




Choose those aspects of curves where $K > 0$.

$\therefore$ There is some value of K large enough where fixed point becomes unstable $\Rightarrow$ limit cycle limit cycle / oscillation





What are the design principles for in biomolecular circuits?

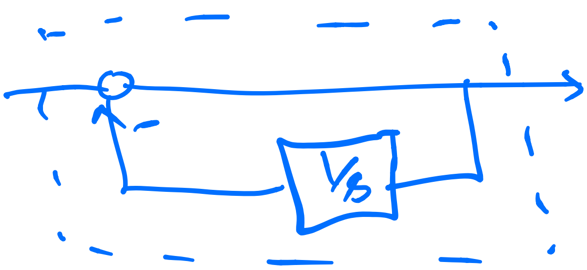
Homeostasis

- temperature
- blood pressure, glucose, iodine, . . .

↳ feedback (integral)

↳ feedforward

What happens when feedback has integrator?



$$\frac{s}{s+1}$$

for small $j\omega$,

→ derivative