26.02.2020

ELL 707

Homeostasis / Robustness

Adaptation.

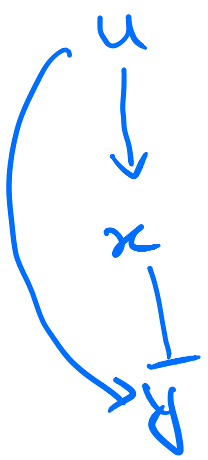
Systems Biology

↓ systems level principles
- integral feedback
- feedforward
- . . .

→ applications + what can we learn

Project ??

Example : Feedforward Loop (FFL)
(Incoherent)



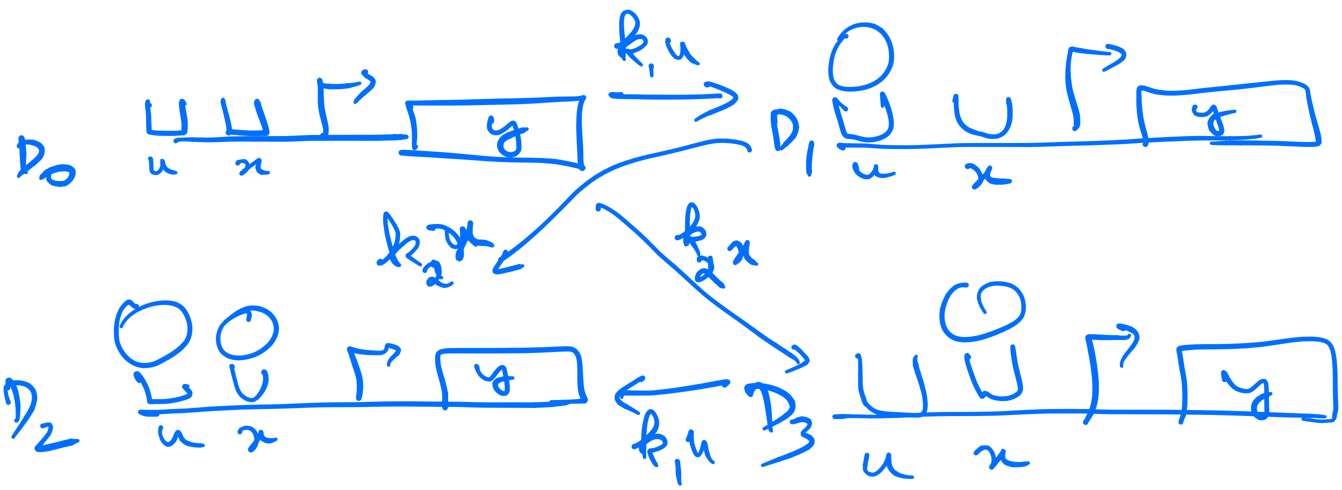
$$\frac{dx}{dt} = \alpha_x u - r_x x$$

$$\frac{dy}{dt} = \ ? \ - r_y y$$

? $\rightarrow \alpha_y\left(u + \frac{K}{K+x}\right)$ logical OR

$\rightarrow \alpha_y\left(u \cdot \frac{K}{K+x}\right)$ ✓ logical AND

Exercise: What kind of expressions are possible?



$$D_0 + D_1 + D_2 + D_3 = D_T$$

$$\begin{tikzcd}
D_0 \arrow[r, shift left, "k_1 u"] \arrow[d, shift right, "k_2 x"'] & D_1 \arrow[l, shift left, "k_-"] \arrow[d, shift left, "k_2 x"] \\
D_3 \arrow[u, shift right, "k_-"'] \arrow[r, shift left, "k_1 u"] & D_2 \arrow[l, shift left, "k_-"] \arrow[u, shift left, "k_-"]
\end{tikzcd}$$

Use mass action (and suitable assumptions) to find D_0, D_1, D_2, D_3 as a function of D_T and other parameters (u, x, k_1, k_2, k_-).

$$\frac{dx}{dt} = \alpha_x u - r_x x$$

$$\frac{dy}{dt} = \alpha_y u \cdot \underbrace{\frac{K}{K+x}}_{\frac{K}{x}} - r_y y$$

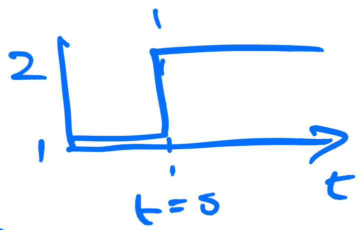
$\frac{K}{x}$, when $x \gg K$

$$\frac{dx}{dt} = \alpha_x u - r_x x$$

$$\frac{dy}{dt} = \alpha_y u \frac{K}{x} - r_y y$$

Q1. Can we exactly solve it?

u = step function



Q2. Steady-states, $\frac{dx}{dt} = 0 = \frac{dy}{dt}$

$$\Rightarrow x^* = \frac{\alpha_x}{r_x} u, \quad y^* = \frac{\alpha_y u}{r_y} \frac{K}{\alpha_x u / r_x} = K \cdot \frac{r_x}{r_y} \cdot \frac{\alpha_y}{\alpha_x}$$