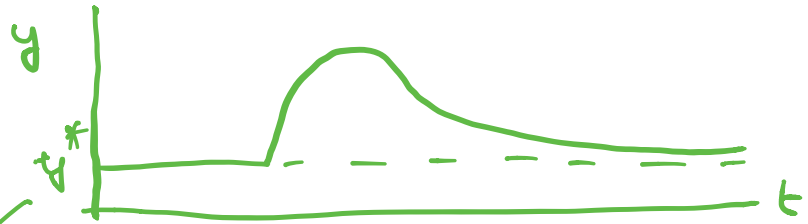
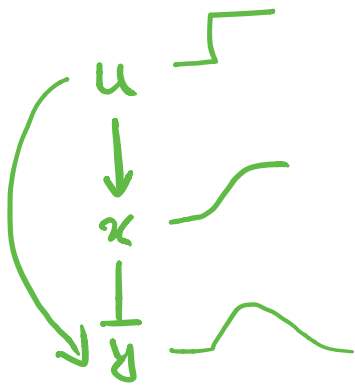


02.03.2020

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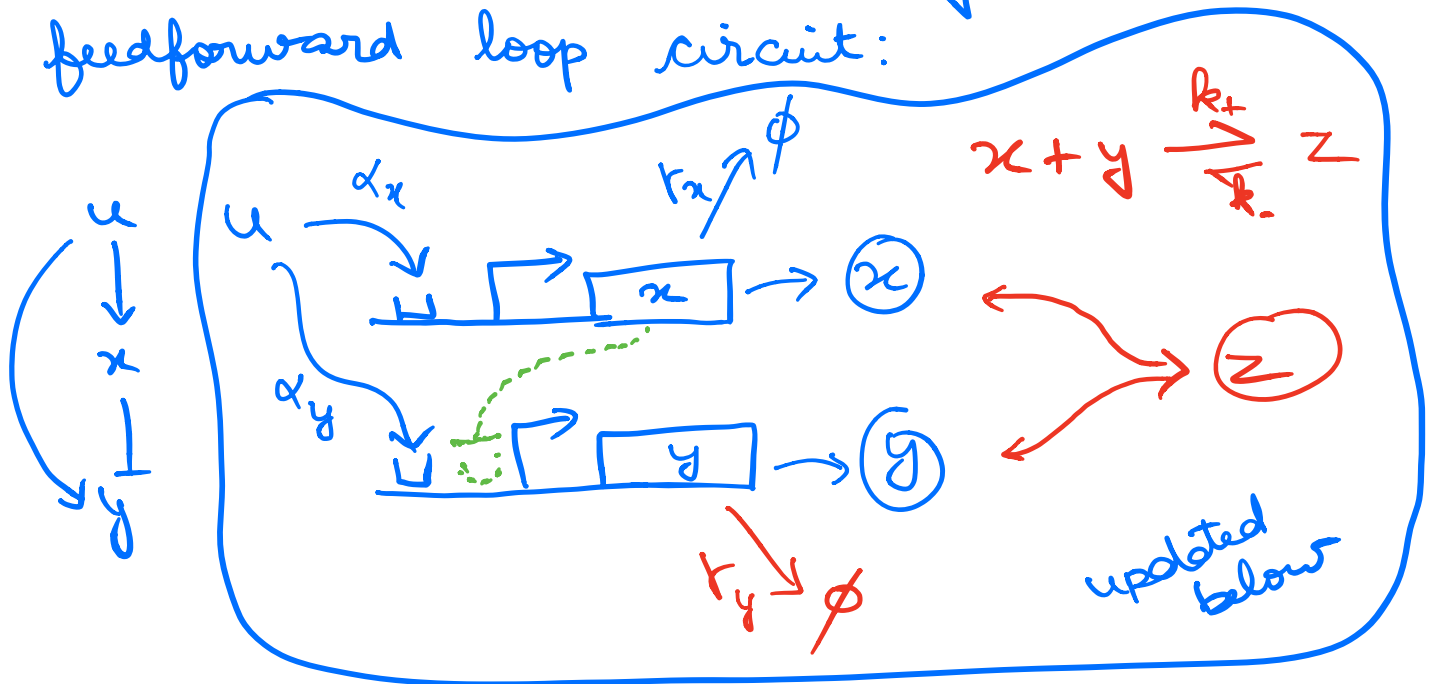
$$y(t) = \frac{\alpha_y}{\alpha_x} K + \frac{\alpha_y}{\alpha_x} K \frac{p-1}{p} e^{-rt} \ln(1 - p + p e^{rt})$$

[for same degradation, $\tau_x = \tau = \tau_y$]

What features of this response are interesting from functional/biological/mathematical point of view?

- The steady-state value of y is independent of input u .
- Transient response has maximum i.e. shows pulse dynamics.
- ...

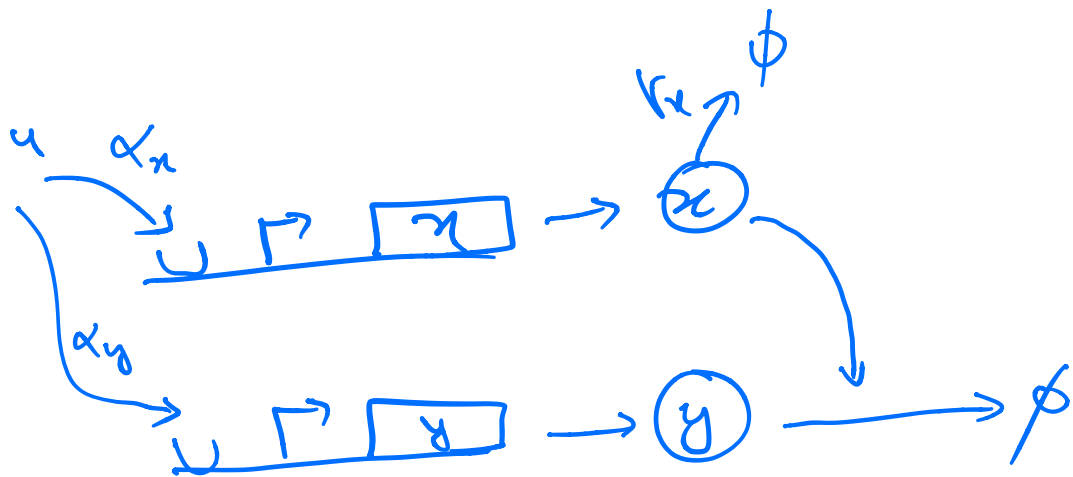
Another implementation of this feedforward loop circuit:



$$\frac{dx}{dt} = \alpha_x u - r_x x$$

this is how $x \rightarrow y$

$$\frac{dy}{dt} = \alpha_y u - R x y$$



$$x + y \Rightarrow [R \cdot x] \Rightarrow x$$

Refer DM 2.4 for more details on this
usually neglected

$$\frac{dx}{dt} = \alpha_x u - \tau_x x$$

$$\frac{dy}{dt} = \alpha_y u - k x y$$

- Steady-state: independence of u ?
- its stability?
- exact solution?

Steady-state: $x^* = \frac{\alpha_x u}{\tau_x}$, $y^* = \frac{\alpha_y}{\alpha_x} \frac{\tau_x}{k}$

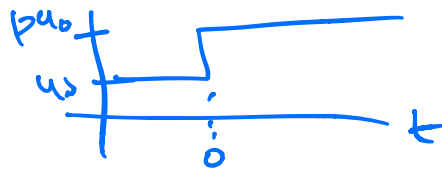
↑
independent of u

- Stability: Calculate linearization

$$J = \begin{bmatrix} -\tau_x & 0 \\ -k y^* & -k x^* \end{bmatrix} \quad (\text{Try!!})$$

⇒ stable?

- exact solution



$$u = \begin{cases} u_0, & t < 0 \\ p u_0, & t \geq 0 \end{cases}$$

$$x(0) = \frac{\alpha_x u_0}{\tau_x}$$

$$y(0) = \frac{\alpha_y}{\alpha_x} \frac{\tau_x}{k}$$

$$\Rightarrow x(t) = x(0) e^{-\tau_x t} + p \cdot x(0) (1 - e^{-\tau_x t})$$

$$\frac{dy}{dt} = \alpha_y \cdot p \cdot u_0 - k x y$$

How to solve?