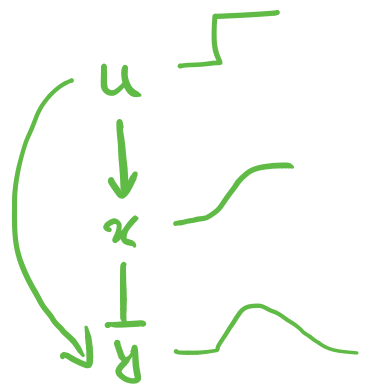
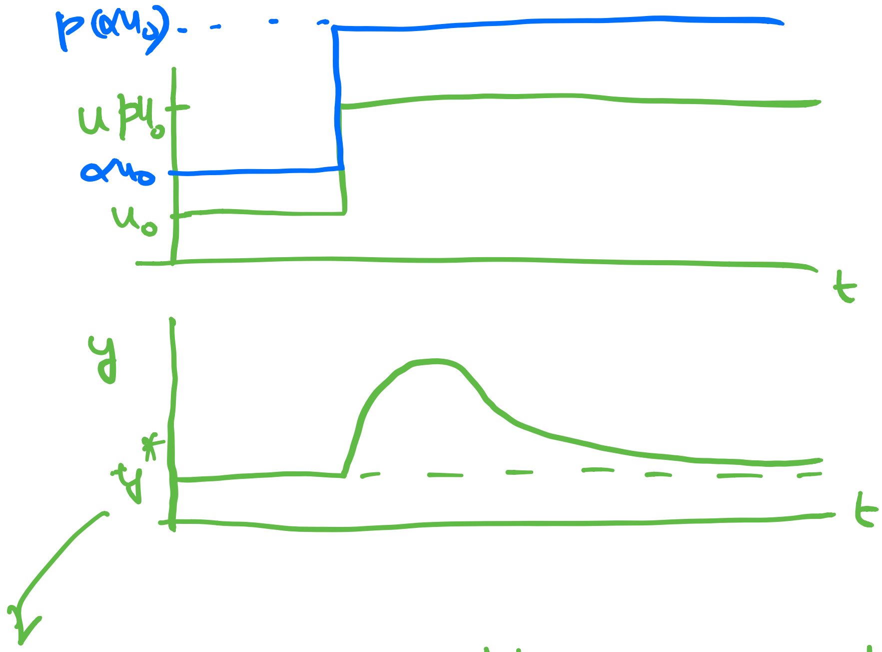
04.03.2020

ELL707





$$y(t) = \frac{\alpha_y}{\alpha_x} K + \frac{\alpha_y}{\alpha_x} K \frac{p-1}{p} e^{-rt} \ln(1-p+pe^{rt})$$

[for same degradation, $r_x = r = r_y$]

This exhibits a fold-change property in the sense that the transient response depends only on 'p' and not the absolute value 'u_0'.

Definition of Fold-Change Detection

Consider a system with input 'u' and output 'y'. System is initially at rest $y(0) = y_0$.

Fold change detection means that the response $y(t)$ is exactly the same for two inputs $u_1(t)$ and $u_2(t)$ that are related by a fold-change $u_2(t) = \alpha\, u_1(t)$, $\alpha > 0$ is constant.

$u_1(t) \longrightarrow y_1(t)$

$u_2(t) \longrightarrow y_2(t)$

If $u_2(t) = \alpha u_1(t) \Rightarrow y_1(t) = y_2(t)$.

Theorem: Consider a system

$$\dot{x} = f(x, y, u) \qquad u - \text{input}$$
$$\dot{y} = g(x, y, u) \qquad y - \text{output}$$

A sufficient condition for this to have the Fold. Change Detection property is

a) it has a stable steady-state

b) this steady-state doesn't depend on input (homeostasis /exact adaptation)

c) for $\alpha > 0$ (homogeneity)

$$f(\alpha x, y, \alpha u) = \alpha f(x, y, u)$$
$$g(\alpha x, y, \alpha u) = g(x, y, u)$$

Proof: We want to prove that

$u_1(t) \longrightarrow y_1(t)$

$u_2(t) \longrightarrow y_2(t)$

If $\alpha u_1(t) = u_2(t) \Rightarrow y_1(t) = y_2(t)$

(for any $\alpha > 0$)

Condition a)+b) $\Rightarrow \quad y_{1,ss} = y_{2,ss}$

u_1

$$\frac{dx_1}{dt} = f(x_1, y_1, u_1)$$

$$\frac{dy_1}{dt} = g(x_1, y_1, u_1)$$

u_2

$$\frac{dx_2}{dt} = f(x_2, y_2, u_2)$$

$$\frac{dy_2}{dt} = g(x_2, y_2, u_2)$$

if $u_2(t) = \alpha u_1(t)$

$$\Rightarrow \quad \frac{dx_2}{dt} = f(x_2, y_2, \alpha u_1)$$

$$\frac{dy_2}{dt} = g(x_2, y_2, \alpha u_1)$$

Normalize x_2: $\quad x_2' = \dfrac{x_2}{\alpha}$

$$\Rightarrow \quad \frac{dx_2'}{dt} = \frac{1}{\alpha}\frac{dx_2}{dt} = \frac{1}{\alpha} f(\alpha x_2', y_2, \alpha u_1)$$

$$= \frac{1}{\alpha} \cdot \alpha f(x_2', y_2, u_1)$$

$$\frac{dy_2}{dt} = g(\alpha x_2', y_2, \alpha u_1)$$

$$= g(x_2', y_2, u_1)$$

$$\Rightarrow \quad \left.\begin{aligned} \frac{dx_2'}{dt} &= f(x_2', y_2, u_1) \\ \frac{dy_2}{dt} &= g(x_2', y_2, u_1) \end{aligned}\right\} \text{same as}$$

Equations are same. If initial conditions are same then $y_1(t) = y_2(t)$

$y_1(0) = y_2(0)$ since independent of input.

$x_1(0) \stackrel{?}{=} x_2'(0) = \frac{x_2(0)}{\alpha}$ will check.

Initially

$f(x_1(0), y_1(0), 0) = 0 = g(x_1(0), y_1(0), 0)$

$f(x_2'(0), y_2(0), 0) = 0 = g(x_2'(0), y_2(0), 0)$

Quiz 6.

1. Consider the system

$$\frac{dx}{dt} = u - x$$

$$\frac{dy}{dt} = 2u - x - y$$

input : u
output : y
internal variable : x

a) Find steady-state output.
b) Is steady-state output independent of u?
c) Is it stable?
d) Does this system exhibit Fold Change Detection?