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"... it is hard to imagine that the observer designed for a known input can serve to estimate the state of the process for the purpose of generating the control input."

- Friedland

MODEL
$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

CONTROLLER

$$u = -\overset{\text{gain}}{K}\,\hat{x}$$

→ forced to use the state estimate

OBSERVER

$$\dot{\hat{x}} = (A - \overset{\text{gain}}{L}C)\,\hat{x} + B\underline{u} + Ly$$

need the input here that depends on the state/estimate

$$\dot{x} = Ax + Bu \quad \quad u = -K\hat{x} = -K(x-e)$$

$$= (A-BK)x + \boxed{BKe}$$

extra term, not previously encountered in state feedback design

$$\dot{e} = \dot{x} - \dot{\hat{x}}$$

$$= \{Ax + Bu\}$$

$$- \{(A-LC)\hat{x} + Bu + Ly\} \quad \quad y = Cx$$

$$= Ax - (A-LC)\hat{x} - LCx$$

$$= (A-LC)x - (A-LC)\hat{x}$$

$$= (A-LC)(x-\hat{x}) = (A-LC)e$$

$$\dot{x} = (A - BK)x + BKe$$

$$\dot{e} = (A - LC)e$$

$$\underset{2n\times 1}{\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix}} = \underset{2n\times 2n}{\left[\begin{array}{c|c} A-BK & BK \\ \hline 0_{n\times n} & A-LC \end{array}\right]} \underset{2n\times 1}{\begin{bmatrix} x \\ e \end{bmatrix}}$$

What is the characteristic polynomial of this system?

Eigenvalues?

$$\det\left(sI - \left[\begin{array}{c|c} A-BK & BK \\ \hline 0 & A-LC \end{array}\right]\right)$$

$$= \det\left(\begin{bmatrix} sI-(A-BK) & -BK \\ 0 & sI-(A-LC) \end{bmatrix}\right)$$

better explanation?

$$= \det(sI-(A-BK))\det(sI-(A-LC))$$

$$= \left\{\begin{array}{c}\text{eigenvalues for} \\ \text{state feedback}\end{array}\right\} \cup \left\{\begin{array}{c}\text{eigenvalues for} \\ \text{observer}\end{array}\right\}$$

even though there is the extra term

Separation Principle ↑

- Design Controller & observer separately
- Then combine them

How to choose eigenvalues? gains /

- Controller : design specifications, input saturation
- Observer : $e \to 0$, $\mathrm{Re}\{\mathrm{eig}\{A-LC\}\} < 0$

  noise in output measurement

- Choice of observer eigenvalues given controller eigenvalues?

Problem:

$$\dot{x} = x + u, \quad x \in \mathbb{R}, \ y \in \mathbb{R}, \ u \in \mathbb{R}$$

$$y = x$$

$k = 1 - \lambda_c$

- design control $u = -kx$ so that eigenvalue is at $\lambda_c < 0$
- design observer with eigenvalue $\lambda_o < 0$ to obtain state estimate $\hat{x}$.
- For $u = -k\hat{x}$, calculate $x(t)$ and $e(t)$?