ELL333 - Multivariable Control

Semester I, 2022-23

Question 1:

Suppose $\dot{x} = Ax + Bu$ and an invertible co-ordinate transformation $z = Px$ is taken to obtain $\dot{z} = \hat{A}z + \hat{B}u$.

a. Find $\hat{A}$ and $\hat{B}$.
b. Show that the system controllability remain invariant under such transformation.
c. For the output equation $y = Cx$, calculate the output matrix after the above mentioned transformation. Comment about the system observability after such transformation.

Question 2:

For the bridge circuit shown in Figure 1

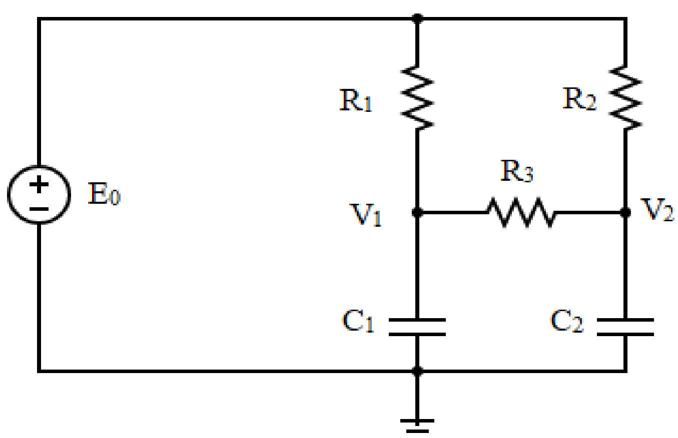

Figure 1

a. Find the state space model with $\begin{bmatrix} V_1 & V_2 \end{bmatrix}^T$ as states and E_0 as input.
b. For balanced bridge condition $R_1C_1 = R_2C_2$, discuss the controllability of the state-space model.

Question 3:

For square matrices A and B, prove that

$$e^A e^B = e^{(A+B)}$$

iff A and B commute, i.e., $AB = BA$.

Question 4:

"If the zero-pole cancellation occurs in the transfer function $H(s)$ of the system, then the system is either uncontrollable or unobservable or both."

Confirm the above statement for the system represented by:

$$H(s) = \frac{s+3}{(s+1)(s+2)(s+3)}$$

Also comment about the controllability and observability of the reduced order system due to the pole-zero cancellation.

Question 5:

For the linear system,

$$\dot{x}(t) = Ax(t)$$

we have the following details

$$x(t) = \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix} \text{ when } x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$
$$x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} \text{ when } x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Find e^{At} and A.

Question 6:

For the op amp circuit shown in Figure 2

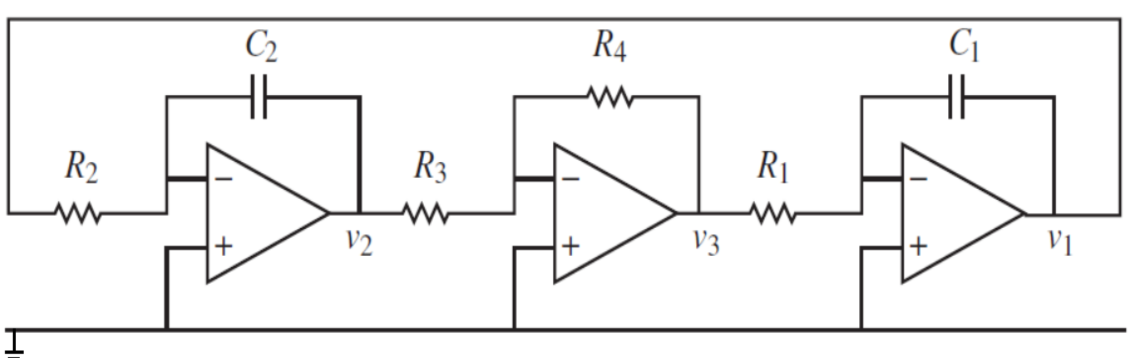

Figure 2

Find the state-space model with v_1 and v_2 as the state variables.