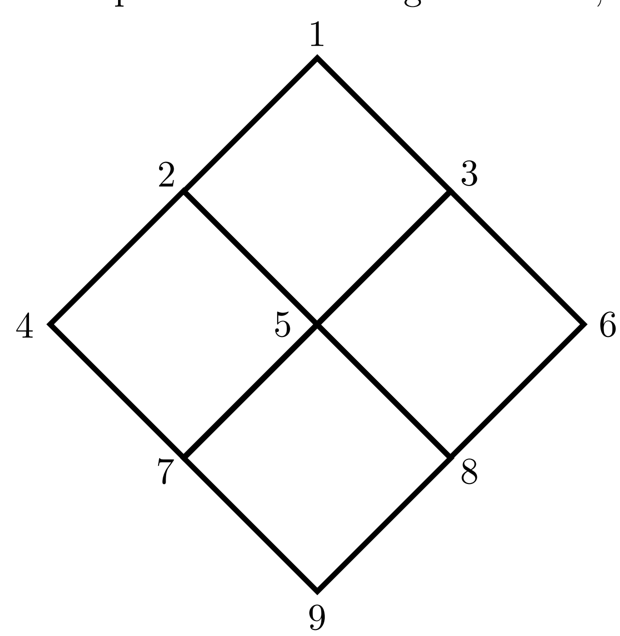
ELL703 > MidSem Exam

Kindly note

\# Duration = 2 hours.
\# Maximum marks = 25.
\# First three questions are for $5\frac{2}{3}$ marks each.
\# Last two questions are for 4 marks each.

1. Find the control sequence $\{u_0, u_1\}$ that minimizes the cost $J = (x_2 + 2)^2 + u_0^2 + u_1^2$ for the scalar system $x_{k+1} = \frac{1}{2}x_k + u_k$, $x_0 = 10$. Find the resulting sequence $\{x_1, x_2\}$ and the minimum value of J.

2. It is desired to go from node 1 to node 9 in the minimum time. Denote t_{ij} as the time to travel from node i to node j, where $i, j \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and t_{ij}^* as the minimum value of t_{ij}. Travel is restricted from the top to the bottom at each time step. For the values given below, find t_{19}^*. Justify your answer.



$t_{12} = 4$
$t_{13} = 5$
$t_{24} = 3$
$t_{25} = 5$
$t_{35} = 3$
$t_{36} = 5$
$t_{47} = 7$
$t_{57} = 8$
$t_{58} = 6$
$t_{68} = 9$
$t_{79} = 2$
$t_{89} = 3$

3. Show that the minimum cost $J^*(t, x_t) = \min_u\{\Phi(T, x_T) + \int_t^T L(\tau, x, u)d\tau\}$ under the constraint $\dot{x} = f(t, x, u)$ satisfies the Hamilton-Jacobi-Bellman Equation, $-\frac{\partial J^*}{\partial t} = \min_{u_t}\left\{L(t, x, u) + \left(\frac{\partial J^*}{\partial x}\right)^T f\right\}$.

4. It is desired to design a slide for children's playground with a shape that minimizes the time it takes to slide from the top of the slide to its bottom. Assume that there is no friction and that the motion is only under the influence of earth's gravity. Using dynamic programming, formulate the problem mathematically. How would you solve this equation?

5. Using dynamic programming, minimize $\sum_{i=1}^n x_i^2$ subject to $\sum_{i=1}^n x_i = c$.