ELL703 > Problems > Continuous-Time Optimal Control

1. [Lewis] A model of an automobile suspension system is given by $m\ddot{y}+ky=u$ where m is the mass, k is the spring constant, u is the upward force on the frame, and y the vertical position.

To conduct a durability test, we repetitively apply a force $u(t)$ and suddenly remove it until failiure occurs. To compute the force, we can solve the following problem. Fin $u(t$ to move the automobile from $y(0)=0$, $\dot{y}(0)=0$ to a final position of $y(T)=h$, $\dot{y}(T)=0$ at a given final time T. Minimize the control energy $J=\frac{1}{2}\int_0^T u^2dt$.
a. Write the state equation if the state is $x=[y,\dot{y}]^T$.
b. Write the state and costate equations, stationarity condition, and boundary conditions. Eliminate u from the state and costate equations.
c. Solve for the costate in terms of the as yet unkown $\lambda(0)$. Solve for the state in terms of the unkown $\lambda(0$ and the known $x(0)$.
d. Use the boundary conditions to find $\lambda(0$. Let $m=k=1$, $T=2$, $h=-4$ (i.e., down) for the remainder of the problem.
e. Find optimal control and omptimal state trajectory. f. Verify that $x^*(T)-[h,0]^T$ as required.

2. [Lewis] **Optimal Control of a Bilinear System.** Let $\dot{x}=Ax+Dxu+bu$, where $x\in R^n$, $u\in R$, and $J=\frac{1}{2}x'(T)S_Tx(T)+\frac{1}{2}\int_0^T(x'Qx+ru^2)dt$. Show that the optimal control consists of a state-costate inner product. Find state and costate equations after eliminating u. These cubic differential equations are very hard to solve.

3. [Lewis] **Temperature Control in a Room.** It is desired to heat a room using the least possible energy. If $\theta(t)$ is the temperature in the room, θ_a the ambient air temperature outisde (a constant), and $u(t)$ the rate of heat supply to the room, then the dynamics are $\dot{\theta}=-a(\theta-\theta_a)+bu$ for some constants a and b, which depend on the room insulation and so on. Define a reasonable performance index and find the optimal control input.

4. [Lewis] **Open-Loop Control of Motion Obeying Newton's Laws.** A particle obeying Newton's laws satisfies $\ddot{x}=u$, where x is position, $\dot{x}$ is velocity, and u is normalized force. Find an analytical expression for the control required to drive any given $x(0$ to any desired $x(T)$, while minimizing $J(0)=\frac{1}{2}\in_0^T ru^2dt$.

5. [Lewis] **Optimal Feedback Control of a Scalar System.** Let the scalar plant be $\dot{x}=ax+bu$ with the performance index $J(t_0=\frac{1}{2}s_Tx^2(T)+\frac{1}{2}\int_{t_0}^T(qx^2+ru^2)dt$. Find the optimal control when only the initial state is fixed.