ELL703 > Problems > Dynamic Programming > Continuous-time

1. Start from the continuous-time cost function,

$$J(t, x(t), u(\tau)) = \phi(x(T)) + \int_t^T L(\tau, x(\tau), u(\tau)) d\tau,$$

with the constraint $\dot{x} = f(t, x(t), u(t))$. Discretize time in N steps each of width T/N. Derive the Hamilton-Jacobi-Bellman equation as $N \to \infty$.

2. [Lewis] For the scalar system with cost $\dot{x} = x + u$ and the cost,

$$J(t_0, x(t_0), u(\tau)) = \frac{1}{2} x^2(T) + \frac{1}{2} \int_{t_0}^T r u^2(\tau) d\tau,$$

where $\tau \in [t_0, T]$. Find $u^*(t)$ that minimizes this cost.

3. Solve the above problem by discretizing time. Numerically compute the optimizing input solution and compare with the analytical solution obtained in the above problem.