ELL703 > Problems > Dynamic Programming > Discrete-time

1. [W. L. Brogan] Consider the scalar system $x_{k+1} = x_k + u_k$ with boundary conditions $x_0 = 0$ and $x_3 = 3$. Find the controls u_0, u_1, and u_2 that minimize $J = \sum_{k=0}^{2}\{u_k^2 + 1\}$. (*Hint: The constraint* $x_3 = 3$ *imposes a unique value on* u_2*.)*

2. [W. L. Brogan] Find the control sequence which minimizes $J = [x_2 + 2]^2 + \sum_{k=0}^{1} u_k^2$ for the scalar system $x_{k+1} = \frac{1}{2}x_k + u_k$, $x_0 = 10$, no restrictions on u_k. Also find the resulting sequence x_k and the minimum value of J.

3. Using the Principle of Mathematical Induction, prove that the optimal cost at each time-step of the discrete-time LQR problem is a quadratic function of the cost.