ELL703 > Problems > Discrete-Time Optimal Control

1. [Lewis] **Optimal Control of a Bilinear System.** Let the scalar plant $x_{k+1} = x_k u_k + 1$ have performance index $J = \frac{1}{2}\sum_{k=0}^{N-1} u_k^2$, with final time $N = 2$. Given x_0, it is desired to make $x_2 = 0$.
a. Write state and costate equations with u_k eliminated.
b. Assume the final costate λ_2 is known. Solve for λ_0, λ_1 in terms of λ_2 and the state. Use this to express x_2 in terms of λ_2 and x_0. Hence find a quadratic equation for λ_2 in terms of initial state x_0.
c. If $x_0 = 1$, find the optimal state and costate sequences, the optimal control, and the optimal value of the performance index.

2. [Lewis] **Control of a Scalar System.** Let $x_{k+1} = 2x_k + u_k$.
a. Find the homogeneous solution x_k for $k = 0, 5$ if $x_0 = 3$.
b. Find the minimum-energy control sequence u_k required to drive $x_0 = 3$ to $x_5 = 0$. Check your answer by finding the resulting state trajectory.
c. Find the optimal feedback gain sequence K_k to minimize the performance index $J_0 = 5x_5^2 + \frac{1}{2}\sum_{k=0}^{4}(x_k^2 + u_k^2)$. Find the tesulting state trajectory and the costs to go J_k^* for $k = 0, 5$.