ELL703 > Problems > Static Optimization with Constraints

1. [Lewis] **Meteor Closest Point of Approach.** A meteor is in a hyperbolic orbit described with respect to the earth at the origin by

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

Find its closest point of approach to a satellite that is in such an orbit that it has a constant position of (x_1, y_1). Verify that the solution indeed yields a minimum.

2. [Lewis] **Geometric Mean Less Than or Equal to Arithmetic Mean.**
(a) Show that the maximum value of $x^2y^2z^2$ on the sphere $x^2 + y^2 + z^2 = r^2$ is $(r^2/3)^3$.
(b) Show that the maximum value of $x^2+y^2+z^2$ on the surface $x^2y^2z^2 = (r^2/3)^3$ is r^2.
(c) Generalize (a) or (b) and so deduce that, for $a_i > 0$,

$$(a_1 a_2 \cdots a_n)^{1/n} \leq (a_1 + a_2 + \cdots a_n)/n$$

3. [Brogan/] Find the control sequence $\{u_0, u_1\}$ that minimizes the cost $J = (x_2 + 2)^2 + u_0^2 + u_1^2$ for the scalar system $x_{k+1} = \frac{1}{2}x_k + u_k$, $x_0 = 10$. Find the resulting sequence $\{x_1, x_2\}$ and the minimum value of J. Use the method of Lagrange Multipliers.

4. [Lange] Find the rectangular box in R^3 of greatest volume having a fixed surface area.

5. [Lange] Let $S(0, r) = \{x \in R^n : ||x|| = r\}$ be the sphere of radius r centered at the origin. For $y \in R^n$, find the point of $S(r)$ closest to y.

6. [Lange] **Projection onto a Hyperplane.** A hyperplane in R^n is the set of points $H = \{x \in R^n : z^*x = c\}$ for some $z \in R^n$ and scalar c. Assume $||z|| = 1$. Find the point on h that is closest to a point $y \in R^n$.

7. [Lange] **Eigenvalues of a Symmetric Matrix.** Let $M = (m_{ij})$ be an $n \times n$ symmetric matrix. Note that M has n real eigenvalues and n corresponding orthogonal eigenvectors. What would the stationary values of $L(x, \lambda) = x^*Mx + \lambda(||x||^2 - 1)$ represent?