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Abstract

In energy-constrained cognitive radio networks (CRNs), the choice on single channel access versus switched multichannel access is critical for energy saving and sustainable network operation. In this paper, we study and compare the energy efficiency of switched (probabilistic) multichannel access (pMCA) and fixed single-channel access (SCA) in CRNs. In pMCA, a secondary user (SU) switches channel with certain probability whenever it encounters a busy channel. In SCA on the other hand, the SU stays on the same channel for its usage and waits for its availability. Via an analytical framework we derive the channel utilization and energy efficiency of the two schemes. From the results we examine the primary user (PU) traffic dependent optimum switching probability in pMCA and the regime of PU activity dynamics where SCA outperforms pMCA.

Index Terms

Cognitive radio, switched multichannel access, single-channel access, DTMC, energy efficiency

I. INTRODUCTION

In cognitive radio networks (CRNs), the secondary users (SUs) make use of the temporal and spectral vacant spectrum spaces in the licensed bands. SUs transmit only when there is an absence of primary user (PU) activity over the channel and cease their operation whenever the PUs activity is detected. A CRN could comprise of multiple SUs operating over multiple PU licensed channels [1]. In such a network, SUs contend among themselves for PU channels to opportunistically transmit their data. In a centralized CRN, central controller allocates the PU channels to the SUs, while in a distributed CRN, a SU listens to the control packet transmissions from the other SUs to find who is going to access the channel. In a dynamic PU traffic, the

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channel state at the time of sensing and the time the SU finally starts its transmission over the channel may be totally different [2]. Hence, we consider that the channels are allocated to the SUs without sensing the channel first. The SUs are considered equipped with in-built sensing capability to exploit vacant spectrum spaces and the SUs operate on a single PU channel at a time. To avoid channel access conflict, a PU channel is allocated to one user at a time. A SU could be allocated a dedicated single channel [3]–[5] or a set of dedicated multiple channels [6]–[8] for their operation.

In a scenario where an SU is allocated a single channel for its transmission (called, single-channel spectrum access (SCA)), typically, the SU opportunistically exploits the channel whenever it senses the channel as idle. If the channel is sensed busy, the SU carries out the sensing process at regular intervals until the PU channel becomes inactive. On the other hand, if an SU is allowed switch-access to dedicated multiple PU channels, it chooses to switch the channel with a certain probability whenever the current one is found busy (called, switched (probabilistic) multichannel spectrum access (pMCA)) [6].

In energy constrained battery operated SUs, e.g., CR sensor nodes, lifetime of the SU nodes is critical. SUs need to be energy-efficient for their prolonged operation in the network [9], [10]. SUs operating over multiple channels incur an additional energy consumption in channel switching as compared to the single channel operation. Recent works [6], [11], [12] indicate that there is a significant amount of energy consumption and delay in channel switching by the SUs. In [6], the authors have shown that the channel switching may result in energy consumption from 2 mJ to 40 mJ, depending on the distance of the channel to switch to. In [11], the authors have assumed 1000 mW as the channel switching power consumption. Additionally, the channel switching delay in channel switching ranges from 150-200 µs depending on specific hardwares [13], [14]. The authors in [12] have considered 1 ms/10 MHz as the SU channel switching delay. As discussed, the channel switching causes significant energy consumption and time delay to the SUs operating over multiple channels that cannot be neglected.

A. Scope and contributions

We consider practically implementable single-channel and switched multichannel access schemes for the SU. The access schemes are optimized with respect to the SU transmission length which ensures that the SUs meet an acceptable PU collision ratio threshold. Performance of the two schemes are computed in terms of SU channel utilization and energy efficiency metrics. SU
channel utilization is the fraction of time the SU transmission is successful. Energy efficiency is the successful SU transmission per unit energy consumption. SCA does not incur channel switching related energy consumption, however the SUs have to wait for the channel availability – which reduces the utilization. It is evident that, there exists a tradeoff between utilization and energy in pMCA and SCA schemes. In this paper, we study this utilization-and-energy tradeoff in the two schemes and investigate the regimes of SU operation where SCA is beneficial over pMCA at different PU traffic intensities and number of PU channels.

SU utilization and energy efficiency of the two schemes with the given PU traffic parameters are derived using discrete-time absorbing Markov chain models. A few recent works in the literature (e.g., [15]–[17]) have utilized discrete-time Markov chains in their performance studies. Our proposed approach stands uniquely in that, (a) considering the correlated PU channel states between the time SU switches to another channel and the time it re-enters the same channel, we develop a novel iterative approach to obtain the steady state PU channel idling probability; (b) via transform domain analysis, we obtain a closed-form expression of pmf of the elapsed time between two successive SU channel access.

Since in this work our primary objective is to study the impact of channel switching related relative performance of pMCA and SCA, a PU channel is assumed to be allocated to one user at a time. This consideration helps in avoiding additional complexity in performance analysis on account of channel access conflict. If the channel access policy in pMCA and SCA is such that there could be channel access conflict, it is expected to similarly affect pMCA as well as SCA. A typical energy-constrained sensor node operating over multiple PU channels does not have multiband channel sensing/access capabilities (e.g., www.zigbee.org). Accordingly, simultaneous multiband sensing/access in a multichannel scenario is out of scope of this work as it does not involve channel switching related issues.

Our numerical results demonstrate that, while pMCA generally offers a higher channel utilization performance, under certain PU activity regimes, SCA has the advantage of higher energy efficiency. It may also be noted that, while pMCA provides a higher channel utilization, SCA scheme can be implemented using lesser hardware and network operating cost. This study is expected to have a significant impact on the system design and network operation of the energy-constrained CR sensor nodes.

A few studies have been reported along the line of our proposed work. The impact of channel switching overhead in CRNs was studied in [18], where the authors computed the
maximum time an SU should remain on a channel before switching. However, channel switching energy consumption and delay was not taken into account. The authors in [19] investigated the access schemes for the single-channel as well as multichannel scenarios, and compared their performance in terms of average SU transmission time and average SU waiting time before it transmits. Energy related aspects were not discussed in their work. In [20], the SU’s channel sensing time was optimized to maximize the energy efficiency and throughput of SUs. Optimal switching strategy was studied in [21], [22] for multiple SUs operation over multiple PU channels. A survey on channel selection in opportunistic spectrum access was presented in [23]. Channel sensing sequence was investigated in [24] for maximizing the average SU throughput.

The current work closest to the study in [25], where SU utilization and energy efficiency tradeoff in single-channel and multichannel access schemes were investigated. However, it did not take into account probabilistic nature of channel switching, heterogeneous PU traffic across different PU channels, SUs channel switching sequence, and channel sensing imperfections.

B. Paper organization

The rest of the paper is organized as follows. The next section details the analytic formulations of the two schemes. Section III presents the results. The paper is concluded in Section IV.

II. SYSTEM MODEL AND PERFORMANCE ANALYSIS

Consider an SU operating in pMCA scheme is allocated $N$ PU channels for its operation. All channels have equal bandwidth. An SU can access a single channel at a time. PU activity over the channels is modeled as an ON-OFF process with the ON (‘Busy’) and OFF (‘Idle’) periods exponentially distributed with respective means $\mu_i$ and $\lambda_i$ over the $i$th channel [6]. PU

Fig. 1: Illustration of $p$MCA.
activity across different channels is considered uncorrelated. SU senses the channel over time $T$. The channel sensing errors are in terms of probability of false alarm ($p_f$) and probability of mis-detection ($p_m$). An SU packet is lost when it collides with a PU packet. Time is divided into slots of duration $T$ (same as sensing duration).

The performance of $p$MCA and SCA schemes are measured in terms of two metrics, namely, SU channel utilization and energy efficiency. SU can operate on a PU channel only when an acceptable PU collision ratio is ensured. PU collision ratio is defined as the ratio of the number of PU transmissions collided with the SUs transmission to the total PU transmissions.

A. Probabilistic multichannel access scheme ($p$MCA)

In $p$MCA, SU starts by operating over the $i$th channel. If the SU senses the channel idle in a slot (the channel may be idle and SU senses it correctly or the channel may be busy and SU mis-detects it as idle), it transmits over the channel in next $l_i$ consecutive slots. The transmission duration ($l_i$ slots) is so adjusted that the PU collision ratio threshold is met. After the transmission, the SU again senses the channel in the following slot to check for the channel availability. If the channel is again found idle, SU transmits in a similar way. However, if the channel is sensed busy (the channel may be busy and SU senses it correctly or the channel may be idle and SU raises a false alarm), SU makes a decision to switch to the next channel or to remain in the same channel. SU operating on the $i$th channel switches to the next channel with probability $\xi_i$. If the SU chooses to remain on the same channel, it senses the channel again in the following slot. The process is repeated. Fig. 1 illustrates this scheme.

As the PU activity over the different channels are different, SU choosing the next channel to switch to in a round-robin fashion may not be optimal. Let $M$ be the transition probability matrix for channel switching sequence, with $M(i, j)$ denoting the probability that the SU operating over channel $i$ chooses to operate next over channel $j$ in an event of channel switching.

\[
\begin{align*}
&\text{OFF} & & \text{ON} \\
&e^{-\frac{T}{\mu_i}} & & e^{-\frac{T}{\lambda_i}} \\
1 - e^{-\frac{T}{\mu_i}} & & 1 - e^{-\frac{T}{\lambda_i}} \end{align*}
\]

Fig. 2: Channel availability representation as an ON-OFF model.
Denoting the ON and OFF channel states as two-state discrete time Markov chain (Fig. 2), the OFF-to-ON state transition probability for the $i$th channel can be computed as:

$$Pr(OFF \to ON) \triangleq P_i(1, 2) = \int_0^T \frac{1}{\lambda_i} e^{-x/\lambda_i} dx = 1 - e^{-T/\lambda_i}.$$  

Similarly, the other transition probabilities are obtained. State transition probability matrix $P_i$ of the Markov chain for the $i$th channel is:

$$P_i = \begin{bmatrix} e^{-T/\lambda_i} & 1 - e^{-T/\lambda_i} \\ 1 - e^{-T/\mu_i} & e^{-T/\mu_i} \end{bmatrix}. \tag{1}$$

Referring to Fig. 1, the SU switches to the $i$th channel at time slot $n_1$. To compute the probability mass function (pmf) $G_i(\tau_i)$ of the time $\tau_i$ the SU remains tuned to the $i$th channel before switching to the next, we form a discrete time absorbing Markov chain model (see Fig. 3), with states defined as ‘SU operation/PU status’. There are five states in the Markov chain: ‘Transmit/Idle’, ‘Transmit/Busy’, ‘Sense/Busy’, ‘Sense/Idle’, and ‘Switch’ (absorbing state). SU enters ‘Transmit/Idle’ state when the channel state is idle and SU senses it correctly. Following the sensing decision, SU transmits for the next $l_i$ slots in this state. After the transmission SU senses the channel again for transition to the next state. ‘Transmit/Idle’ state comprise of transmission in the first $l_i$ slots followed by sensing in the last slot. In ‘Sense/Idle’ state, the channel is idle, however, SU senses the channel as busy. Hence, the SU decides to remain idle in the channel and sense the channel again in the next slot. Transition to ‘Switch’ state occurs when the channel is sensed busy by the SU and the SU decides to switch to the next channel. Other transitions are similarly carried out.

Denote $k$-step $i$th channel state transition probability matrix as $P_i^k$. SU transits from ‘Transmit/Idle’ to ‘Sense/Busy’ when the channel is sensed busy in the last slot of ‘Transmit/Idle’ phase and the SU chooses to stay on the current channel. One-step transition probability from ‘Transmit/Idle’ to ‘Sense/Busy’ state is $P_i^{l_i+1}(1, 2)(1-p_m)(1-\xi_i)$. Here, $P_i^{l_i+1}(1, 2)$ denotes the
To obtain correctly (final state ‘Sense/Busy’) and decides to operate over the same channel, the factors transition of channel state from idle to busy in \( l_i + 1 \) slots. As the SU detects the busy channel correctly (final state ‘Sense/Busy’) and decides to operate over the same channel, the factors \((1 - p_m)\) and \((1 - \xi_i)\) are included in the transition probability. Similarly, one-step transition probability from state ‘Transmit/Busy’ to ‘Switch’ is given as \((P_i^{(l_i+1)}(2, 1)p_f + P_i^{(l_i+1)}(2, 2)(1 - p_m))\xi_i\). In this case, the channel may transit from busy to idle, or busy to busy state in \( l_i + 1 \) slots, however, the SU detects the channel busy and chooses to switch.

Computation of performance metric using the pdf of \( \tau_i \), i.e., \( G_i(\tau_i) \) is cumbersome (as would be seen later) due to the convolutions involved in the time domain. Hence, we resort to the computation of probability generating function (pgf) \( G_i(z) \) of \( G_i(\tau_i) \) and is given as:

\[
G_i(z) = \sum_{j=0}^{\infty} z^j G_i(j). \tag{2}
\]

Suppose, upon switching to the \( l_i \)th channel (for example, at \( n_1 \)) the SU finds the channel is available with probability \( \delta_i \). The SU spends \( l_i + 1 \) slots in ‘Transmit’ states (states 1 and 2) \( (l_i \) slots for data transmission and 1 slot for sensing) and 1 slot in ‘Sense’ states (states 3 and 4). To obtain \( G_i(z) \), we denote \( R_i^z \) by (3), \( U_i^z \) by (4), and \( S_i^z \) by (5).

\[
R_i^z = \begin{bmatrix}
  z(1 + 1)P_i^{(l_i+1)}(1, 1)(1 - p_f) & z(1 + 1)P_i^{(l_i+1)}(1, 1)p_m
  
  z(1 + 1)P_i^{(l_i+1)}(2, 1)(1 - p_f) & z(1 + 1)P_i^{(l_i+1)}(2, 1)p_m
  
  zP_i(2, 1)(1 - p_f) & zP_i(2, 1)p_m
  
  zP_i(1, 1)(1 - p_f) & zP_i(1, 1)p_m
\end{bmatrix}. \tag{3}
\]

\[
U_i^z = \begin{bmatrix}
  z\delta_i(1 - p_f) & z(1 - \delta_i)p_m & z(1 - \delta_i)(1 - p_m)(1 - \xi_i) & z\delta_ip_f(1 - \xi_i)
\end{bmatrix}. \tag{4}
\]

\[
S_i^z = \begin{bmatrix}
  z(1 + 1)P_i^{(l_i+1)}(1, 1)p_f + P_i^{(l_i+1)}(1, 2)(1 - p_m)\xi_i
  
  z(1 + 1)P_i^{(l_i+1)}(2, 1)p_f + P_i^{(l_i+1)}(2, 2)(1 - p_m)\xi_i
  
  zP_i(2, 1)p_f + P_i(2, 2)(1 - p_m)\xi_i
  
  zP_i(1, 1)p_f + P_i(1, 2)(1 - p_m)\xi_i
\end{bmatrix}. \tag{5}
\]

Note that the transition probabilities in the transient states (states A, B, C, and D) is given as \( V_i = R_i^{(z=1)} \). \( U_i^{(z=1)} \) is the initial state probabilities of the transient states and the elements of \( S_i^{(z=1)} \) give the transition probability from transient state to ‘Switch’ state. The powers of \( z \) in these matrices denote the number of slots SU operated on each of the states. The fundamental matrix \( W_i = (I_i - V_i)^{-1} \) gives the expected number of visits to a transient state starting from another state (before being absorbed in ‘Switch’ state). \( I_k \) is a \( k \times k \) identity matrix. Define \( \Omega_i \)
as the probability of last slot of the \(i\)th channel being idle (\(\Omega_i(1)\))/busy (\(\Omega_i(2)\)) before switching to the next channel (at \(n_2\)) as:
\[
\Omega_i = U_i^{(z=1)}W_iF_i
\]  
where
\[
F_i = \begin{bmatrix}
P_i^{(i+1)}(1,1) p_f \xi_i & P_i^{(i+1)}(1,2)(1-p_m) \xi_i \\
P_i^{(i+1)}(2,1) p_f \xi_i & P_i^{(i+1)}(2,2)(1-p_m) \xi_i \\
P_i(1,1) p_f \xi_i & P_i(1,2)(1-p_m) \xi_i \\
P_i(2,1) p_f \xi_i & P_i(2,2)(1-p_m) \xi_i \\
\end{bmatrix}
\]

where the first (respectively, second) column of \(F_i\) gives the probability of switching from the transient state when the last slot is idle (respectively, busy).

Using (3), (4), and (5), \(G_i(z)\) in (2) is obtained as:
\[
G_i(z) = z(1 - \delta_i) \xi_i + \sum_{j=0}^{\infty} U_i^z (R_i^z)^j S_i^z = z(1 - \delta_i) \xi_i + U_i^z (I_4 - R_i^z)^{-1} S_i^z. \tag{7}
\]

First factor \(z(1 - \delta_i) \xi_i\) indicates that the channel was found busy at \(n_1\) and the SU decided to switch the channel, while the second factor indicates the probability that the SU switches to the next channel after operating over at least one of the states in the absorbing Markov chain.

For each PU channel, the probability generating function of \(T_i\) (i.e. \(G_i(z)\)) is obtained. The unknown variables are \(\delta_i\) (the probability of finding the channel idle while SU switches to the \(i\)th channel). Next we show how these \(\delta_i\) are computed.

Referring to Fig. 1, the SU switches from the \(i\)th channel at time \(n_2\) and returns to it at time \(n_4\). During this interval (elapsed time) \(\sigma_i \triangleq n_4 - n_2\), SU operates over other channels. SU switches to the other channels according to the channel switching sequence matrix \(M_i\). Expected number of visits to a channel \(j\) before the SU switches back to channel \(i\) (denoted by \(\beta_i(j)\)) is computed next. Consider an absorbing Markov chain with absorbing state as channel \(i\) while the transient states are channels \(j = \{1, 2, \cdots, i-1, i+1, \cdots, N\}\). Transition probability in the transient states for this Markov chain \(M_i\) is given as \(M\) with all elements of the \(i\)th column of \(M\) replaced by 0. This states that the \(i\)th channel becomes the absorbing state. The fundamental matrix of this absorbing Markov chain is
\[
B_i = (I_N - M_i)^{-1}.
\]

The \((i, j)\)th element of \(B_i\) gives the expected number of visits to channel \(j\) between two successive visits to channel \(i\) i.e., \(\beta_i(j) = B_i(i, j)\).
Pmf of $\sigma_i$ (denoted as $H_i(n)$) is the convolution of all $G_j(n)$ weighed with their expected number of visits to state $j$ before the SU switches back to channel $i$. Hence $H_i(n) = h_i(n - (1 + \sum_{j \neq i} \beta_i(j)) n_s)$, where $h_i(n)$ is,

$$h_i(n) = G_i^{\ast \beta_i(1)} \otimes \cdots \otimes G_i^{\ast \beta_i(i-1)} \otimes G_{i+1}^{\ast \beta_i(i+1)} \otimes \cdots \otimes G_N^{\ast \beta_i(N)}.$$  

Here $G_j^{\ast \beta_i(j)}$ is $\beta_i(j)$ times convolution of $G_j$ and $\otimes$ denotes convolution. The factor $(1 + \sum_{j \neq i} \beta_i(j)) n_s$ is the number of slots used up in channel switching. Pgf of $H_i(n)$, $\mathcal{H}_i(z)$ is:

$$\mathcal{H}_i(z) = z^{(1 + \sum_{j \neq i} \beta_i(j)) n_s} \prod_{j \neq i} \mathcal{G}_j(z)^{\beta_i(j)}.$$

The probability $\delta_i$ in steady state for the $i$th channel is obtained as:

$$\delta_i = \sum_{j=0}^{\infty} H_i(j)(\Omega_i(1)\mathbf{P}_i^j(1,1) + \Omega_i(2)\mathbf{P}_i^j(2,1)). \quad (8)$$

Define $a_i = \mathbf{P}_i(1,1)$ and $b_i = \mathbf{P}_i(2,2)$. Thus,

$$\delta_i = \frac{\Omega_i(1)(1 - b_i)}{(2 - a_i - b_i)}(\mathcal{H}_i(z = 1) + \frac{(1 - a_i)}{(1 - b_i)}\mathcal{H}_i(z = (a_i + b_i - 1)) + \frac{\Omega_i(2)(1 - b_i)}{(2 - a_i - b_i)}(\mathcal{H}_i(z = 1) - \mathcal{H}_i(z = (a_i + b_i - 1)).$$

Note that $\mathcal{H}_i(z)$ depends on $\mathcal{G}_j(z)$ which in turn depends on $\delta_j$, $\forall j \in \{1, 2, \cdots, N\}, j \neq i$. Hence, $\delta_i$ depends on all $\delta_j$. $\delta_i$ in steady state is obtained by iterating (8). We start with an initial value of $\delta_i$ for all channels. At each step of iteration, $\delta_i$ for all channels are updated using (8). After sufficient iterations, the values of $\delta_i$ converge to their steady state values.

Once we obtain $\delta_i$ in steady state for all channels, the expected number of slots $E_i$ the SU remains on the $i$th channel before switching to the next is given as

$$E_i = (Y_i(1) + Y_i(2))(l_i + 1) + Y_i(3) + Y_i(4), \quad (9)$$

where $Y_i = U_i^{(z=1)}W_i$ gives expected number of visits to transient states before being absorbed. Expected number of slots SU transmitted successfully per $i$th channel visit instance is given as
\(l_iY_i(1)e^{-l_iT/\lambda_i}\), where the probability of SU successful transmission in \(l_i\) slots is \(e^{-l_iT/\lambda_i}\). SU channel utilization and energy efficiency are obtained as:

\[
\mathcal{U}_{pMCA} = \frac{l_iY_i(1)e^{-l_iT/\lambda_i} + \sum_{j \neq i} \beta_i(j)l_jY_j(1)e^{-l_jT/\lambda_j}}{E_i + n_s + \sum_{j \neq i} \beta_i(j)(E_j + n_s)},
\]

\[
\mathcal{E}_{pMCA} = \frac{l_iY_i(1)e^{-l_iT/\lambda_i} + \sum_{j \neq i} \beta_i(j)l_jY_j(1)e^{-l_jT/\lambda_j}}{Y_i(1) + Y_i(2)(\Phi_{se} + l_i\Phi_{tx}) + (Y_i(3) + Y_i(4))\Phi_{se} + \Phi_{sw} + n_s\Phi_{id}} + \sum_{j \neq i} \beta_i(j)[(Y_j(1) + Y_j(2))(\Phi_{se} + l_j\Phi_{tx}) + (Y_j(3) + Y_j(4))\Phi_{se} + \Phi_{sw} + n_s\Phi_{id}]
\]

where \(\Phi_{se}\), \(\Phi_{tx}\), and \(\Phi_{id}\) are respectively the SU energy consumption in channel sensing, data transmission, and idling per slot. Channel switching incurs \(\Phi_{sw}\) amount of energy consumption per switch along with the idling energy consumption for the switching duration. The computation for \(\mathcal{U}_{pMCA}\) and \(\mathcal{E}_{pMCA}\) can be carried out for any \(i\).

### B. PU Channels with i.i.d. PU activities

We now consider a special case where the PU activity across all channels are i.i.d. For this case, we drop the subscript \(i\) denoting the \(i\)th channel from the previous section as all channels are identical. PU ON and OFF periods are distributed exponentially with means \(\mu\) and \(\lambda\) respectively. The probability of channel switching is considered equal to \(\xi\) across all channels. In this scenario, we consider two cases of choosing the next channel in case of channel switching event. In the first case, the next channel to switch to is chosen randomly with all channels equiprobable, while in the second case the next channel is chosen in a round robin fashion.

From Fig. 1, the SU switches to the \(i\)th channel at \(n_1\). Pf of \(\tau\), the SU remains on the channel before switching to the next is given by (7). Elapsed time between successive \(i\)th channel access instances is given as \(\sigma\). Pf and Pf of \(\sigma\) is given as \(H(n)\) and \(H(z)\).

In the case where the SU switches channel randomly, the pmf of \(H(n)\) is given as:

\[
H(n) = \sum_{j=1}^{\infty} D(j)G^n(G^i(n - (j + 1)n_s)).
\]

The probability \(D(j)\) that the SU returns to the same channel after \(j\) switchings is found as:

\[
D(j) = \left(\frac{N - 2}{N - 1}\right)^j \frac{1}{N - 2}.
\]
In this case, the pgf $\mathcal{H}(z)$ of $H(n)$ is obtained as:

$$\mathcal{H}(z) = \frac{1}{N - 2} \sum_{j=1}^{\infty} \left( \frac{N - 2}{N - 1} \right)^j z^{(j+1)n_s} G(z)^j$$

$$= \frac{z^{2n_s} G(z)}{N - 1 - z^{n_s} G(z)(N - 2)}$$

For round robin channel switching, the pmf of $\sigma$ is $H(n) = G^{(N-1)}(n-Nn_s)$, as the SUs goes around all the channels sequentially and comes back again to the $i$th channel after $N$ switchings.

Pgf of $H(n)$, $\mathcal{H}(z)$ is given as $\mathcal{H}(z) = z^{Nn_s} G(z)^{(N-1)}$.

The probability $\delta$ in steady state is obtained by iterating

$$\delta = \sum_{j=0}^{\infty} H(j)(\Omega(1)P^j(1,1) + \Omega(2)P^j(2,1)),$$

where $\Omega$ is obtained from (6). Define $a = P(1,1)$ and $b = P(2,2)$. Thus,

$$\delta = \frac{\Omega(1)(1-b)}{(2-a-b)} (\mathcal{H}(z = 1) + \frac{(1-a)}{(1-b)} \mathcal{H}(z = (a+b-1))$$

$$+ \frac{\Omega(2)(1-b)}{(2-a-b)} (\mathcal{H}(z = 1) - \mathcal{H}(z = (a+b-1))).$$

Note that in this case, $\delta$ is updated once every iteration. Once we obtain $\delta$ in steady state, the expected number of slots $E$ the SU remains on a channel before switching is obtained from (9). Expected number of slots SU transmitted successfully per channel visit instance is given as $lY(1)e^{-IT/\lambda}$, where the probability of SU successful transmission in $l$ slots is $e^{-lT/\lambda}$. SU channel utilization and energy efficiency are obtained as:

$$\mathcal{U}_{pmca} = \frac{lY(1)e^{-IT/\lambda}}{E + n_s}$$

$$\mathcal{E}_{pmca} = \frac{lTY(1)e^{-IT/\lambda}}{[(Y(1) + Y(2))(\Phi_{se} + l\Phi_{tx}) + (Y(3) + Y(4))\Phi_{sw} + \Phi_{sw} + n_s\Phi_{i}]}.$$
C. Single-channel access (SCA)

In SCA, a single channel is available at the SU for its operation. When the channel is sensed busy, the SU senses the channel at every following slot until the channel is sensed idle. SU transmits for $l$ consecutive slots whenever the channel is sensed idle. Fig. 4 illustrates this access scheme.

We construct a four state discrete time Markov chain to compute the SU channel utilization and energy efficiency as shown in Fig. 5. Channel states are combination of PU channel state and SUs operation state as were stated in $pMCA$ analysis. If the channel is idle and sensed by SU as idle, then the Markov state is ‘Transmit/Idle’. Similarly the other states are defined.

Denote $X$ as one-step state transition probability matrix of this Markov chain. Transition probability from state ‘Transmit/Busy’ to ‘Sense/Idle’ is obtained as $X(2,4) \triangleq P(l+1)(2,1)p_f$. The other probabilities are similarly computed. $X$ is given in (15).

$$X = \begin{bmatrix}
    P(l+1)(1,1)(1-p_f) & P(l+1)(1,2)p_m & P(l+1)(1,2)(1-p_m) & P(l+1)(1,1)p_f \\
    P(l+1)(2,1)(1-p_f) & P(l+1)(2,2)p_m & P(l+1)(2,2)(1-p_m) & P(l+1)(2,1)p_f \\
    P(2,1)(1-p_f) & P(2,2)p_m & P(2,2)(1-p_m) & P(2,1)p_f \\
    P(1,1)(1-p_f) & P(1,2)p_m & P(1,2)(1-p_m) & P(1,1)p_f
\end{bmatrix}. \quad (15)$$

Limiting probability $\pi$ of the Markov chain is obtained using the relation $\pi = \pi X$. SU channel utilization and energy efficiency are given as:

$$U_{SCA} = \frac{l\pi(1)e^{-lT/\lambda}}{l(\pi(1) + \pi(2)) + 1}, \quad (16)$$

$$E_{SCA} = \frac{l\pi(1)e^{-lT/\lambda}}{l\Phi_{tx}(\pi(1) + \pi(2)) + \Phi_{se}}. \quad (17)$$

The SU transmission length $l$ over a PU channel is constrained by the acceptable PU collision ratio $\eta$ of the system. Assume that the collision caused to PU spans at most one PU ON duration, i.e., $l \ll \mu/T$. Collisions over a PU ON-OFF cycle happen when the SU mis-detects the busy

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**Fig. 5:** Markov chain representation of SU states for SCA scheme.
channel as idle and transmits and when the PU initiates a transmission in between the SU’s transmission. Expected number of PU slots collided with SU transmission in a PU ON period is $l p_m \mu / (l p_m + 1)$. Here, $l p_m / (l p_m + 1)$ is the fraction over which the SU transmitted in the PU ON period. Similarly, expected number of slots collided in case of PUs reappearance during SUs transmission is given as $(1 - p_f) E[\text{SU transmission collision} | \text{transmission collided}]$.

Optimal SU packet length $l$ is obtained as:

$$
(1 - p_f) \sum_{k=1}^{l} \frac{(l - k + 1) e^{-\frac{T(l-k)}{\lambda}} (1 - e^{-\frac{T}{\lambda}})}{1 - e^{-\frac{T}{\lambda}}} + \frac{l \mu}{T(l + \frac{1}{p_m})} \leq \frac{\eta \mu}{T}
$$

or,

$$
\frac{l \mu p_m}{T(1 + l p_m)} + \frac{(1 - p_f)(l - a_l - a - a(l+1))}{(1 - a(l+1))(1 - a)} \leq \frac{\eta \mu}{T}.
$$

The above expression for $l$ is a non-linear equation and close form expression cannot be obtained. LHS in (18) is an increasing function of $l$. Hence, optimal $l$ is the maximum $l \in \mathbb{N}^+$ for which (18) is satisfied. Upper bound on $l$ is $\eta \mu / p_m (T \mu - \eta \mu)$. Bisection method is used to obtain optimal $l$.

Recall that, the effect of channel sensing duration $T$ on the sensing imperfection has been incorporated in our analysis in terms of mis-detection probability $p_m$ and false alarm probability $p_f$. The effect of optimized $T$ on the SU performance is not investigated in our study. Because the sensing duration in both the pMCA and SCA are considered equal, the effect of sensing duration on their performance is similar. Therefore, any change in sensing duration has no effect on the conclusion of our study. Interested readers are referred to [26]–[28] on sensing duration optimization studies.

### III. Numerical results

The performance of the two schemes with respect to the two metrics, namely, utilization and energy efficiency of SUs are presented in this section. We consider slot duration $T$ to be 50 $\mu$s. SU power consumption in data transmission, channel sensing, and idling are respectively 1980 mW, 1320 mW, and 990 mW [29]. Default channel switching energy $\phi_{sw} = 2$ mJ [6] and switching delay is taken as $n_s = 2$ slots [13]. Though the probability of false alarm and mis-detection depend on the channel condition between the PU and the SU as well as the channel sensing duration of the SU, for demonstration purpose, we consider probability of false alarm to be $p_f = 0.02$ and probability of mis-detection to be $p_m = 0.01$ [30]. With a different set
of values of $p_f$ and $p_m$, the SU performance could be similarly obtained. PU collision ratio threshold $\eta = 0.1$. Simulations are carried out in Matlab.

We start by studying the performance of the two schemes over channels with i.i.d. PU activity. PU average OFF duration $\lambda = 10$ ms and average ON duration $\mu$ is varied to achieve different PU activity dynamics. Fig. 6 shows the relative performance of SCA and $p$MCA with channel switching probability $p_s = \{0.3, 0.9\}$ and $N = \{2, 4\}$, at different values of PU activity duty cycle $\rho = \frac{\mu}{\lambda + \mu}$. Round robin channel switching is employed in all cases unless otherwise specified.

We observe that the channel utilization is low for low PU activity duty cycle $\rho$ and it initially increases with $\rho$. This is because, at low $\mu$ (i.e., low $\rho$), acceptable number of PU collision slots is low, which induces small SU packet length $l$. Hence, the channel utilization is low in this case. As $\rho$ increases, $l$ increases, thereby increasing the utilization. However, at further higher values of $\rho$, the channel is busy most of the time, hence the channel utilization by the SU decreases.

The achievable utilization is high in $p$MCA at higher values of $\xi$ because the SU gets more opportunity to transmit by frequently switching the channels. At lower $\mu$ (i.e., lower $\rho$), the channels are idle most of the time. Hence, waiting on the same channel is beneficial to channel switching in this case. At a higher value of $\rho$, the channel is busier than the idle period. SU switches channel more often in this regime, causing a higher energy consumption. Hence, the energy-efficiency of $p$MCA $E_{pMCA}$ is lower than the energy efficiency of SCA, $E_{SCA}$ at low $\rho$. This explains the cross-overs points on energy efficiency performances of SCA and $p$MCA as the PU channel activity is varied.

Fig. 7 shows the effect of round-robin and random channel switching on SUs performance
Fig. 7: Channel utilization and energy efficiency versus PU activity duty cycle for round robin (RR) and Random channel switching schemes. \( \lambda = 10 \text{ ms}, \ N = 5, \ \eta = 0.1, \) and \( \Phi_{sw} = 2 \text{ mJ}. \) sim: simulation, ana: analysis.

**Motivated by the above observation, the rest of the results, we use round robin channel switching in the pMCA scheme.**
In Fig. 8, we present SU channel utilization and energy efficiency versus number of channels $N$ at different values of PU activity duty cycle $\rho$ and with channel switching probability $\xi = 0.5$. $N = 1$ corresponds to the SCA scheme, while pMCA scheme correspond to $N > 1$. With the increase in $N$, SU channel utilization increases because more number of channels provide a higher transmission opportunity to the SUs. However, the utilization saturates to the asymptotic value as the number of channels increases to a large value. Similar observation is made from the energy efficiency metric variation with $N$. Energy efficiency of SCA scheme surpasses that of pMCA scheme at low values of $N$. This is because, channel switching energy consumption adds to the energy budget of the SUs in pMCA and lowers the energy efficiency. At high values of $N$, high utilization compensates the channel switching related consumption and thereby increases the energy efficiency of the SUs. As the number of channels $N$ is increased in pMCA, energy efficiency saturates to an asymptotic value, because SU returns to the same channel after a long time and the probability of channel availability after a long interval tends to $\frac{E[OFF]}{E[OFF]+E[ON]}$.

Fig. 9 presents the effect of channel switching delay $n_s$ on the SUs performance metrics for $\xi = 0.5$ and $\lambda = 10$ ms. Increase in $n_s$ has two-fold effect of low channel utilization (more slots are used up in channel switching) and low switching energy consumption per slot ($\Phi_{sw}/n_s$). Depending on the factor that dominates, energy efficiency varies with the change in $n_s$. For low $\mu$ (i.e., $\rho$), finding transmission opportunity on channel switching is higher decreasing energy efficiency of SUs for higher $n_s$. At higher $\mu$, channels are busy most of the time reducing loss of transmission opportunity for increased $n_s$. Hence $\Phi_{sw}/n_s$ dominates over reduced channel utilization increasing energy efficiency for higher $n_s$. For higher $n_s$, low utilization dominates
PU activity duty cycle, $\rho$

Utilization

$pMCA \Phi_{sw} = 2\text{mJ}, \eta = 0.05$

$pMCA \Phi_{sw} = 4\text{mJ}, \eta = 0.05$

$pMCA \Phi_{sw} = 2\text{mJ}, \eta = 0.1$

$pMCA \Phi_{sw} = 4\text{mJ}, \eta = 0.1$

SCA $\eta = 0.05$

SCA $\eta = 0.1$

Fig. 10: Effect of PU collision ratio threshold $\eta$ and SU channel switching energy consumption $\Phi_{sw}$ on SU channel utilization and energy efficiency for $\lambda = 10$ ms, $\xi = 0.5$ and $N = 4$.

which finally leads to decrease in energy efficiency in all cases.

The effects of PU collision ratio threshold $\eta$ and SU channel switching energy consumption $\Phi_{sw}$ on the channel utilization and energy efficiency are shown in Fig. 10. A higher value of $\eta$ ensures a high utilization because high PU collision ratio threshold permits larger packet size transmissions. Higher $\Phi_{sw}$ increases the SU energy consumption, resulting in lower $E_{pMCA}$.

Fig. 11: (a) Energy efficiency versus channel switching probability $\xi$ for $\lambda = 10$ ms and $\rho = 0.5$; (b) energy efficiency of $pMCA$ with optimal $\xi$ and SCA at different $\rho$ with $\lambda = 10$ ms and $\Phi_{sw} = 2$ mJ.

The effect of channel switching probability $\xi$ on SU energy efficiency $E_{pMCA}$ at different values of $N$ is shown in Fig. 11a. At a lower value of $\xi$ the SU remains on the same channel most of the time – reducing its transmission opportunity over other channels, whereas at a higher $\xi$ the channel is switched at most instances when the channel is found busy, increasing SU energy consumption. Clearly, there exists an optimal $\xi$ ($\xi^{opt}$) for the maximum energy efficiency. With larger value of $N$, switching to a different channel increases the probability of finding an available channel. Hence, $\xi^{opt}$ increases with $N$. 
Fig. 12: SU channel utilization and energy efficiency with $N = 3$ PU channels and heterogeneous PU activity. Scenario 1: $\lambda = \{10, 10, 10\}$ ms, and $\mu = \{5, 10, 15\}$; Scenario 2: $\lambda = \{5, 10, 15\}$ ms, and $\mu = \{10, 10, 10\}$. $\Phi_{sw} = 2$ mJ and $n_s = 2$.

Fig. 11b shows the energy efficiency performance of $pMCA$ (with $\xi_{opt}$) and SCA. To this end, we use bisection method to obtain the optimal switching probability $\xi_{opt}$, because the energy efficiency is a unimodal function of switching probability $\xi$. The figure also shows the $\xi_{opt}$ at different $\rho$. At $\rho$ close to 0 and 1, $\xi_{opt}$ is close to 0. In these cases SCA and $pMCA$ perform equally, since $\xi = 0$ correspond to SCA. At moderate values of $\rho$, $pMCA$ performs better than the SCA.

Next, we consider PU channels with heterogeneous PU activity. Two scenarios are studied with 3 PU channels. In Scenario 1, PU activity parameters are $\lambda = \{10, 10, 10\}$ ms and $\mu = \{5, 10, 15\}$ ms for respectively the first, second, and third channel, while for Scenario 2, the parameters are $\lambda = \{5, 10, 15\}$ ms and $\mu = \{10, 10, 10\}$ ms for the three channels. Fig. 12 plots the utilization and energy efficiency of the SUs. In $pMCA$ scheme, the next channel to switch to is chosen with equal probability. The plot also shows metric for $\xi_{opt}$ $pMCA$, where the probability of channel switching over each channel is optimized to obtain maximal energy efficiency.

From the utilization plot, we observe that the $pMCA$ offers higher utilization to the SUs as compared to the SCA schemes, however, the energy efficiency of SCA is higher than $pMCA$ for some channels. As compared to the $pMCA$, SCA over channel with higher $\lambda$ and lower $\mu$ have better energy efficiency performance. Simulations were carried out to obtain the optimal switching probability in $pMCA$ schemes for optimal energy efficiency performance. $\xi_{opt}$ in Scenario 1 is $\{0.12, 0.05, 0.16\}$ while for Scenario 2 is $\{0.07, 0.06, 0.01\}$. The energy efficiency performance of $\xi_{opt}$ $pMCA$ is slightly better than the SCA schemes as the channel switching probabilities are closer to 0 in both the cases.
IV. CONCLUSION

In this paper, we have investigated the tradeoff associated with probabilistically switched multichannel access (pMCA) in CRNs in terms of channel utilization by the SU and its energy efficiency. The tradeoff performance of pMCA has been contrasted with the single-channel access (SCA) scheme under different PU channel activity dynamics. In the pMCA scheme, PU activity and number of switched channels dependent optimum channel switching probability has been observed. Our study has demonstrated that, while pMCA has the advantage of higher channel utilization, under certain PU traffic conditions SCA outperforms pMCA on energy efficiency. In these regimes, especially for energy-constrained cognitive channel access, it is beneficial to use SCA, as it reduces the complexity and cost of network implementation as well as channel sensing hardware in the SUs. As an extension of the work, multiuser CR MAC protocol exploiting the tradeoffs with SCA and pMCA in a dynamic PU traffic scenario can be developed for optimal energy efficient operation.

REFERENCES


