Exploiting multiple description coding for intermediate recovery in wireless mesh networks

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A B S T R A C T

Guarding against channel errors in wireless networks has been a challenging research problem, especially when transmitting time-constrained contents, like streaming video or image. Multiple description coding (MDC) is an effective means to combat bursty packet losses over wireless channels, and it is especially promising for the applications where retransmission is unacceptable. In MDC, a frame is broken into several equally important descriptions which can be sent over multiple paths to the destination. Meshed multipath routing over wireless mesh networks and MDC, when combined together, could make video/image transmission more error-resilient by providing opportunities to recover erroneous video packets at the intermediate nodes along the route.

In this work, we propose to combine MDC and meshed multipath routing in such a way that multiple descriptions are sent over different paths in the network and are merged at the intermediate nodes for possible recovery of corrupted descriptions. The routing uses the idea that when multiple descriptions join at intermediate points it can help in partial recovery of lost or corrupted descriptions by using the link error information. This approach reduces the possibility of error propagation, thereby improving the overall image/video quality at the destination. In order to achieve high gain in terms of peak signal-to-noise ratio (PSNR), we investigate an optimum spatial interleaving based MDC scheme that maximizes the intermediate recovery possibility for a given recovery filter design. We also explore the choice of an optimum number of descriptions for merging at an intermediate merging point that maximizes the PSNR gain. Our simulation study shows that, by introducing recovery possibility even at one intermediate node, it is possible to achieve a PSNR gain of around 5–6 dB using our coding, routing, and recovery strategies.

1. Introduction

Wireless channels are inherently error-prone. Transmission of time-constrained information, like streaming video and images, is challenging because retransmission of a lost or corrupted packet is infeasible. Therefore, the error control measures have to be taken at the coding, routing, and message reconstruction stages.

For image/video transmission, conventionally three types of coding techniques are practiced: non-progressive coding, progressive coding, and multiple description coding (MDC). Non-progressive coding is performed purely for higher compression efficiency, but it is not suitable for delay constrained transmissions. Progressive (layered) coding, on the other hand, works well when packets are sent and received in order without loss. But when a packet is lost, the reconstruction stalls until that particular packet is received. For still image transmission, the most common way of source coding is progressive encoding. Unfortunately, TCP-based content delivery suffers from these pauses during reconstruction, because the delay in receiving a retransmitted packet may be much longer than the inter-arrival time between received packets. While dealing with time-constrained video or image frame transmission over unreliable paths, an effective way to guard against wireless channel errors is to use source diversity where a frame is split into equally important descriptions or message blocks by the source encoder. This is known as MDC. The descriptions in general can be equally important (symmetric MDC) or of unequal importance (asymmetric MDC). Multiple correlated descriptions are generated from a compressed image frame by adding some redundancy to estimate the lost descriptions from the other correctly received descriptions. Thus, in this scheme retransmission is avoided, but at the cost of a reduced compression efficiency. MDC is a more robust technique for wireless environments, as an MDC receiver can decode the data with a low but acceptable quality even if some of the descriptions are lost.

Multipath routing for multiple descriptions makes the transmission process more resilient to errors due to network
congestion and intermittent link failures. This has been studied in the context of video transmission in wireline networks, albeit without considering partial error recovery. In order to mitigate network congestion and intermittent link failure related errors, commonly in wide-area wireline routing MD coded video packets are sent over completely non-overlapping paths, which are both node and edge-disjoint (Alasti et al., 2001). A similar approach has been employed for wireless networks in Gogate et al. (2002). However, the choice of completely disjoint path is debatable in wireless networks, where the predominant cause of corruption is the bursty link errors, rather than nodal congestion. A link-disjoint multipath route would be better suited to guard against link errors, whereas node-overlapped routes would provide more routing flexibility in wireless networks. More importantly, the possibility of some overlapping intermediate nodes and very few or no overlapping links imply that different packets are likely to experience uncorrelated error conditions, which can aid in repairing some corrupted packets along the route as they move toward the destination. In other words, if the meshed multipath structure of the wireless network is exploited, the quality of received image can be improved further.

Most of the earlier works (Goyal, 2001) on MDC for image transmission over wireless networks focus on the various forms of source diversity. When a stream of MD coded packets is divided into substreams and sent over multiple paths, different portions of the frame are protected from link errors by virtue of path diversity. The descriptions are combined at the destination to reconstruct the frame with a better quality compared to the case when the entire stream is sent over a single lossy link. The effect of path diversity along with source diversity for transmission of video or image frames over wireless links was studied in Gogate et al. (2002). However, the benefit of using more than two descriptions along with path diversity was not explored in there. In addition, partially corrupted packets are usually discarded at the link layer. Instead, if the correlation information of partially corrupted packets with correctly received packets are exploited to repair the corrupted packets, overall quality of the reconstructed message is expected to be better.

In this work, we explore two aspects of source diversity and path diversity: (1) splitting a frame suitably into multiple descriptions, and (2) applying a routing strategy that aids partial recovery of descriptions. We propose measures to maximize the quality of a decoded frame via intermediate recovery as follows: If some or all correlated descriptions, which travel via different paths, pass through an intermediate node, there is a high likelihood that a corrupted description can be recovered to a considerable extent at the intermediate stage, before being further corrupted as it arrives at the destination. However, in a network, without a judicious splitting of packets if we want to exploit the path diversity, only very few or none of the correlated descriptions may merge at the intermediate nodes. Thus, achieving an increased gain from error recovery requires a proper spatial distribution of MD coded packets that will allow maximum correlated packets to converge at the intermediate nodes. Moreover, given multiple paths and a potential number of intermediate merging points, the optimum number of descriptions as well as their splits may have an impact on the quality of recovery. In our approach, given a total number of descriptions, we derive an optimal subset of descriptions to be merged at an intermediate node, and identify which streams should be directed towards the merging points. The proposed MDC-aware routing algorithm ensures that the chosen descriptions are directed toward the merge points. Additionally, to maximize the quality of recovery at a merging point we use the link error information of the correlated (neighboring) pixels in estimating a lost or corrupted pixel. A preliminary version of this work was presented in De et al. (2011). To the best of our knowledge, any other study on intermediate recovery of MD coded packets jointly with an enabling routing strategy and link error aware decoding has not been reported yet in the literature. Since our aim in this work is to explore the benefits of joint multipath route aware coding, packet forwarding to enable intermediate recovery, and cross-layer decoding strategy, we have chosen simple spatial interleaving based MDC and a modified 2 × 2 spatial filter technique for decoding. There have been several works reported in the literature on more advanced MDC techniques (Goyal, 2001). Likewise, temporal interpolation for a stream of image/video frames has been exploited. Also, more efficient and complex spatial interpolation techniques are available in the image processing literature (Ramamurthy and Raychaudhuri, 1995; Hemami and Meng, 1995; Wang and Zhu, 1998). To this end, we do not claim to advance the individual MDC and image error recovery techniques. Our main contributions in this paper can be summarized as follows.

- Design of a spatial interleaving based MDC scheme, which is suitable for splitting an image frame into multiple descriptions. The decoding strategy, which uses wireless channel error information, is shown to maximize partial error recovery at the intermediate nodes.
- Design of a routing scheme that is aware of the coding scheme and helps in merging desired descriptions at the intermediate nodes for efficient partial error recovery.

Note that, the focus of our work has been to mainly show the possibility of using multiple descriptions of image/video streams while leveraging multipath routing in wireless network for better QoS. To this end, we show objectively, without necessarily depending on any particular networking technologies, the benefit of our proposed scheme in improving the overall quality of the image/video delivered. As for the implementation of our proposed scheme in IP networks, it is mostly an engineering exercise of bringing together several existing IP networking concepts ranging from UDP Lite to deep packet processing and implementing the proposed scheme. We plan to look into the implementation aspects in our future work.

The paper is organized as follows. In Section 2, we use simple examples to illustrate how partial recovery of packets at intermediate nodes can be more efficient than recovery at the destination. We also describe here how the routing strategy uses the knowledge of coding scheme to help the partial recovery process. In Section 3, we explain the coding and decoding schemes in detail. Section 4 describes the proposed routing scheme to aid intermediate recovery. In Section 5, we evaluate our coding and routing schemes with respect to different tunable parameters. Prior work in this domain is reviewed in Section 6. We conclude in Section 7.

2. Problem formulation

In this section, using simple examples we will illustrate the usefulness of intermediate recovery of packets. Consider a scenario where an image is transmitted from a source node to a destination node over a multihop wireless network. At the source node, an image frame is split into M equally important descriptions that are divided into equal-sized packets for transmission to the destination via multipath routes. In order to extract the best

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1 It is also possible to split a frame into descriptions of varying importance as well as unequal sized packets.
gain in image quality, the routing scheme must try to converge a chosen set of descriptions at the intermediate points.

A wireless link error is modeled in terms of pixel error rate (PER). A noisy link implies high PER and a good link means low PER. In order to compare the relative qualities of the recovered image frames, we use the peak signal-to-noise ratio (PSNR) metric which is defined as:

\[ \text{PSNR} = 20 \log_{10} \frac{255}{\text{RMSE}} \]

where the root mean square error (RMSE) of a reconstructed \( d \times d \) image frame \( f \) with respect to the original frame \( F \) is given by

\[ \text{RMSE} = \sqrt{\frac{1}{d^2} \sum_{i=0}^{d-1} \sum_{j=0}^{d-1} (f(i,j) - F(i,j))^2} \]

and each pixel value is represented as an 8 bit codeword.

To illustrate the objective of the routing scheme, we choose a double diamond shaped network shown in Fig. 1(a), as an example. Half of the links are “good” with a PER of 1% and the remaining ones are “bad” with a PER of 20%. In this example, we code the image into two descriptions, which is the most common MDC strategy. Given the link conditions, one stream faces significant errors and the other one is less corrupted. We compare the image quality for two scenarios: (i) where the image is reconstructed only at the destination and (ii) where the descriptions are also partially recovered at the intermediate node.

Fig. 1. A diamond shaped wireless network with links having high and low PER: (a) single intermediate merging point and (b) two intermediate merging points.

Figures 2(a) and (b) show the recovered image at the destination with and without intermediate recovery, respectively. Figures 2(a) and (b) show the recovered image at the destination with and without intermediate recovery, respectively.

There is a visible improvement in the image quality, which is also indicated by a gain in PSNR. The PSNR of the image with intermediate recovery is 27.26 dB as opposed to 21.13 dB, giving a gain of almost 6 dB.

**Observation 1.** Intermediate recovery with two descriptions shows significant gain in image quality.

In the above example, we considered that all the descriptions are available for recovering the corrupted pixels at the intermediate node. However, this may not be the case always – specially in a large network, as one might be interested in exploiting path diversity to a greater extent, where a larger number of descriptions might offer more error resilience. It may be noted here that, more path diversity and a higher number of descriptions make the routing and forwarding strategies more complex.

In our next example, we consider a network as shown in Fig. 1(b) which has two intermediate merging points. The frame is split into four descriptions, which are transmitted through four different links, with an equal mix of good and bad links. In this case, only two descriptions can meet at each intermediate merging point. Figures 3(a) and (b) show the reconstructed image with and without intermediate recovery, respectively.

Again, a perceptible improvement in image quality is confirmed by about 5 dB gain in PSNR, where the PSNR with intermediate recovery is 26.47 dB and that with no intermediate recovery is 21.55 dB.

**Observation 2.** There is a gain in PSNR by partial recovery of streams, although with a drop in the overall gain, as compared to the case where all descriptions merge at one intermediate node.

The Observation 2 raises a question whether, in a large network, where many edge-disjoint paths can be found, it is a good idea to keep on increasing the number of descriptions. An associated question is, whether there is any impact of the number of merging points on the end-to-end delay performance. We will address these questions in Sections 3 and 4 since it is closely tied to the coding scheme adopted for splitting a stream at the source node. We will show that, increasing the number of descriptions merging at an intermediate node beyond a certain value does not increase the PSNR. This optimal value of the number of descriptions is the primary input for devising a multipath routing strategy.
Given a network topology, our routing scheme proceeds by first finding a set of nodes that can be used as the merging points. Since it is aware of the optimal number of descriptions to be merged, it can direct the source node to create the right number of descriptions ($M$), which is the product of the number of descriptions to be merged at a node ($N$) and the number of merging points ($J$). In the next step, the routing algorithm searches for edge-disjoint paths that can carry the descriptions and also ensures that the chosen subset of descriptions converge at the intermediate nodes. In practice, in a random network, ensuring merger of certain descriptions at specific intermediate points is challenging. As a proof of concept on intermediate recovery, we have considered in this work a regular graph (i.e., a two-dimensional mesh), in which the merging locations can be pre-decided for a given source-destination pair. Figure 4 illustrates the scenario, where we have chosen four merging points along the diagonal of a $4 \times 4$ mesh. Assuming that the number of descriptions to be merged at an intermediate point is two, there will be eight descriptions created at the source. The best routing strategy should find eight link-disjoint paths for the descriptions before they converge in subsets of two at each intermediate merging point. In general, the routing algorithm finds a shortest path for each substream towards its chosen intermediate merging point. We will show that our technique of finding shortest path in small clusters introduces more merging points in addition to the chosen ones, thereby further aiding the intermediate recovery.

### 3. Coding and decoding scheme

In MDC technique an image frame is split into multiple equally important descriptions. A set of pixels from the frame is assigned to each description in a way that ensures if some of the descriptions are corrupted during transmission, the other descriptions can help in recovering the corrupted pixel values. The breakup of an image frame into separate descriptions is called the coding stage, usually performed at the source node. The recovery step is usually performed at the destination node at the time of frame reconstruction. In our recovery approach, in addition to applying the standard filter-based recovery method, we also take advantage of cross-layer information about the link conditions, usually available as physical link error information, to improve the recovered image quality. In this section, we present the coding and decoding approaches in detail.

#### 3.1. Coding scheme

Given an image frame in some compressed form, there are different ways in which the frame can be split into multiple descriptions. If we have to create two descriptions, then a naive way of splitting a frame would be to assign alternate rows of pixels to descriptions 1 and 2. Note that, the key idea behind splitting an image frame into multiple descriptions is to spread the spatial information of the frame in the form of pixel values into different descriptions, so that when one description is corrupted, then the other spatially correlated description can be used to interpolate the corrupted pixel values. To achieve this goal in a 2D scenario, the best possible spatial distribution of descriptions is achieved by assigning alternate pixels to descriptions 1 and 2, as shown in Table 1. This is because, our recovery strategy takes only the North–South–East–West pixels to interpolate a corrupted pixel. In this splitting approach, if a pixel value in description 1 is corrupted, it can be efficiently inferred from description 2, provided description 2 is less corrupted, and vice versa. However, in a large network, intuitively more than two descriptions will
exploit path diversity better and minimize the effects of link errors.

When the number of descriptions are increased, there are many different ways to split the image. We adopt a simple scheme of a fixed value shift in the description id as we move to the subsequent rows in the image. For example, if there are eight descriptions, and we apply a shift of size 2, then the first pixels in rows 1–5 would belong to descriptions 1, 3, 5, 7, and 1, respectively. Note that, finding an optimal shift size at the coding stage and devising a forwarding strategy to enable arrival of a suitable set of descriptions at a merging point are very important.

This optimal shift size at the MD coding stage and judicious forwarding of the descriptions jointly would ensure that the spatially correlated descriptions are not simultaneously corrupted and they have a likelihood of merging at the intermediate nodes to maximize the PSNR gain via intermediate recovery.

We chose the shift size \( \psi \) equal to half the number of descriptions. Thus, for eight descriptions the shift size is 4, for 16 it is 8, and so on. Table 2 shows a 4 \( \times \) 16 image block split into 16 descriptions with \( \psi = 8 \). The choice of the shift size is motivated by the following lemma.

**Lemma 1.** Given a 2 \( \times \) 2 filter and four-description merging, a shift size equal to half the number of descriptions results in the maximum chance of intermediate recovery.

**Proof.** During the recovery of a pixel \( i \), we try to merge the following four descriptions: \( (i+1), (i+x), (i+x+1), \) where \( x \) is an arbitrary shift value. The rationale behind the selection of this target description set is to merge a pair of descriptions which are natural neighbors (e.g., \( i, (i+1) \) and \( (i+x), (i+x+1) \)), with maximum chance of recovery. According to our filter design, we merge the four descriptions corresponding to the four adjacent pixels (\( N, S, E, W \)) to \( i \) (equivalently, the adjacent pixel set comprise \( (i-1), (i+1), (i+x), (i-x) \)). So, we have \( (i+1), (i+x) \) common between the adjacent pixel set and the target description set that we can always use for recovery. Now, if we have \( x = \psi \), then we have \( (i+x) = (i-x) \), and we have an extra description available for recovery from the adjacent pixel set. Hence, we have the maximum chance of recovery when the shift value is \( \psi \).

We illustrate the benefit of grouping four descriptions with an example. Let us assume that for a set of 16 descriptions, we have a four-set as \{1-2-9-10\}, and the corresponding two-sets will be \{1-9\} and \{2-10\}. Assume, the descriptions 1,2,9 are bad and the description 10 is good. Table 3 shows the recovery potential for each description in 2-description merging and 4-description merging. A description has a chance of recovery if at least one of its neighboring pixels is from a good description. Table 3 shows that in 4-description merging we can recover three descriptions, as opposed to just two descriptions in the 2-description merging case. This further strengthens our argument in favor of using a forwarding strategy that aims at merging four descriptions at a merging point.

### Table 2

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**3.2. Cross-layer decoding strategy**

Our recovery scheme is activated at every merging point to enable partial recovery of corrupted descriptions, in addition to the conventional recovery at the destination. In order to recover a pixel \( X \), we take help of the four neighboring pixels: north–south–east–west. If all the descriptions are available during recovery, a naive approach would be to assign equal weight to each of the pixels. However, if one of the descriptions has suffered more corruption, then the probability that it will incorrectly interpolate the pixel-\( X \) increases. In contrast, during the interpolation, we propose to assign weights to the neighboring pixels based on their error rates. Specifically, the weights assigned are inversely proportional to the error rates inferred from physical layer link errors.

In addition to the weighted interpolation approach, we introduce another optimization by leaving those descriptions untouched which have suffered low pixel error rate. This ensures that during interpolation a noisy description does not end up smearing a good description. This cross-layer information link error helps us achieve a higher quality in image reconstruction.

In case of partial recovery, all the neighboring pixels may not be available at an intermediate merging point since we may have to merge as low as two descriptions at an intermediate point. If a neighboring pixel information is unavailable, we distribute the weights to the available pixels based on the technique discussed.

### 4. Multihop routing strategy

We model the network as a graph \( G = (V, E) \), where \( V \) is the set of \( N \) vertices representing the nodes (routers in a multihop wireless network) and \( E \) is the set of edges (or wireless links connecting the routers). The routing problem is to find paths between a pair of nodes \( (s, t) \) such that maximal path diversity can be exploited by the multiple descriptions while introducing suitably selected intermediate wireless error recovery points.

Before we proceed, let us present some concepts used in our routing solution. A *path* in a graph is formally defined as a chain of vertices such that the consecutive pair of nodes define an edge and in which the terminal vertices are distinct. Thus, a path between \( s \) and \( t \) can be represented as \( P_{st} = \{v_1, \ldots, v_l \subseteq V | (v_{l-1}, v_l) \in E, v_1 = s, \ldots, v_l = t \} \).

In a multihop routing scenario, end-to-end transmission delay and path diversity are two critical parameters. In the following lemma we explore the relationship between end-to-end transmission delay and the path diversity in multiple paths used in the transmission.

**Lemma 2.** End-to-end delay of a multihop route decreases with the increase in link disjointedness in the paths and decrease in node overlaps.

**Proof.** For simplicity of the proof consider a regular network topology and imagine any number of descriptions per image frame and any multipath routing combinations are realizable. All nodes are assumed equipped with single transceiver system of...
equal data transmission rate. Let the number of message blocks (number of descriptions) per frame be \( M \) and transmit time of a message block be \( T \) time units. Also, let the number of hops of any route from \( s \) to \( t \) – node and link-disjoint, or link-disjoint with node overlap be \( H \).

First, consider an \( H \)-hop from \( s \) to \( t \) single path route. At each node along the route, the transmission time is \( M \). For \( H \)-hop route, the total end-to-end transmission delay is \( D_{st}^{(H)} \) units.

Now consider disjoint multipath routes. Let, \( M \) describes be equally split into \( K \) substreams, thus each substream having \( M/K \) number of blocks. With \( M = 2^i \), \( n=1,2,\ldots \) to have integer values of \( M/K \), \( K \) can assume values \( K = 2^{n-1} \), where \( i = 1, \ldots, \log_2 M + 1 \). Having single transceiver at each node, the source will incur transmission delay \( D_{st}^{(H)} \) to transmit all \( M \) descriptions to \( K \) outgoing links. Likewise, total delay to receive all \( M \) descriptions at the destination is \( D_{st}^{(H)} \). At any other intermediate node along different disjoint routes, where it carries \( M/K \) blocks, the transmission delay incurred is \( D_{st}^{(H)} \). The route having \( H-2 \) such intermediate hops, the total end-to-end transmission delay over \( K \) disjoint multipath routes, denoted by \( K_{th} \), is \( D_{st}^{(K)} = (2 + ((H-2)/K))MT \) units.

Next, consider \( K \) \( H \)-hop multipath routes, with \( J \) intermediate merging points, denoted by \( K_{th} \). To have identical segments, \( H \) should be an integer multiple of \( 4 \) and \( J \) can assume values \( J = 2^{i-1} - 1 \), where \( i = 1, \ldots, \log_2 H \). Following a similar logic as in disjoint end-to-end routes, the total end-to-end transmission delay can be obtained as:

\[
D_{st}^{(K)} = (J+1) \left( \frac{H}{K} + \frac{2 + ((H-2)/K))}{K} \right) MT.
\]

Note that, \( K=1 \) corresponds to \( D_{st}^{(K)} = HMT \) – the single path routing case, and \( K=M \) and \( J=0 \) corresponds to \( D_{st}^{(K)} = (2 + ((H-2)/K))MT \) – the disjoint multipath routing case. Moreover, \( J = (H/2) - 1 \) corresponds to \( D_{st}^{(K)} = HMT \) – for any value of \( K \), which implies that, end-to-end transmission delay is rather a function of node overlaps along the route (the value of \( J \)); the more the node overlaps, the more the delay.

An example case of end-to-end transmission delay with \( M=8 \), \( H=16 \), and \( T=1 \) (unit delay) is shown in Fig. 5. It is evident from the above observations on delay and Fig. 5 that minimal intermediate merging points \( (J) \) and minimal edge overlap (captured in \( K \)) among the paths between \( s \) and \( t \) ensure minimal end-to-end transmission delay.

However, while it is important to have path diversity to ensure resilience against wireless error, node overlaps among the paths are also required to improve PSNR, where the overlapping nodes can serve as the intermediate recovery points. From the earlier discussion on coding scheme for a particular filter that we used, it is evident that the best chance of partial recovery happens only when it is possible to merge four descriptions at an intermediate node. Thus the two, apparently orthogonal, primary goals of our routing scheme in the light of the above discussions are as follows:

1. Maximize the merging of four descriptions at intermediate nodes for partial recovery. Also, maximize the merging of more than one descriptions at the intermediate nodes, whenever possible.
2. Ensure maximal edge independence (or path diversity) among the paths between \( s \) and \( t \) for wireless error resilience.

**Algorithm 1 (Routing strategy)**

**procedure** FindPaths\((G,s,t)\);

Find \( C \) as the minimum vertex cover of the \( s-t \) cut in \( G \) with maximum maximal matching;

for all \( c \in C \) do

FindEdgeIndependentPaths\((G,s,c)\);

FindEdgeIndependentPaths\((G,c,t)\);
end for

**procedure** FindEdgeIndependentPaths\((G,x,y)\);

\( G_{\text{curr}} = G \);

for \( i = 1 \) to \( 4 \) do

FindShortestPath\((G_{\text{curr}}, x,y)\);
end for

**procedure** FindShortestPath\((G,x,y)\);

Apply a shortest path algorithm to find a path \( P \) in \( G \) between \( x, y \);

Increase the cost for all edge \( e \in P \);

Return \( P \);
4 × |C| descriptions are generated (|C| being the number of anchor nodes discovered) and each group of four related descriptions is merged at an anchor node before splitting them again and forwarding them to the destination (t) node for merging once again. Thus four node overlapping paths are found with minimal edge overlap between the source s and each of the anchor nodes and then again from the anchor nodes to the destination t. Of course merging of the descriptions are done whenever possible at the intermediate nodes other than the anchor nodes and the destination.

Our heuristic selects the maximum of minimal vertex separator\(^3\) for s and t in G (also known as \(s \rightarrow t\) vertex separator) as the anchor nodes, since each path from s to t meets the separator set at some vertex. By Menger’s Theorem, the size of the smallest \(s \rightarrow t\) vertex separator S is exactly the same as the number of vertex disjoint paths from s to t. Finding a vertex separator is an NP-Hard problem (Bui and Jones, 1992), except for some graphs with special properties (e.g., planar graphs) and with some constraints (e.g., finding any vertex separator) there exists linear-time algorithm (Lipton and Tarjan, 1979). For a grid network shown in Fig. 4, the maximum of minimal vertex separator for paths from node 1 to 16 can be easily found as the diagonal consisting of nodes \(4,7,10,13\). The selection of maximum of minimal vertex separator partially contributes to our first routing objective by finding node disjoint paths. Note, as discussed in Section 2, an increase in the number of node overlaps also increases the end-to-end delay. However, in this paper we do not optimize or guarantee end-to-end delay.

We meet the second objective in two ways. The first way is implicit in the derivation of maximum of minimal vertex separator as the minimum vertex cover corresponding to the maximal matching of maximum \(s \rightarrow t\) cut in G. This ensures the accommodation of the maximal number of paths without edge overlap through the vertex separator nodes. This follows from the following two lemmas.

**Lemma 3.** The maximum of minimal vertex separator for s and t in G can be expressed as the minimum vertex cover corresponding to the maximal matching of maximum \(s \rightarrow t\) cut in G.

**Proof.** The minimum vertex cover corresponding to the maximal matching of a \(s \rightarrow t\) cut in G is a minimum vertex separator for the given \(s \rightarrow t\) cut. If we take the minimum vertex cover corresponding to the maximal matching of the maximum \(s \rightarrow t\) cut in G then we get the maximum of minimal vertex separator for s and t in G. □

**Lemma 4.** The maximum number of edge-disjoint paths possible between s and t is the number of vertices in the minimum vertex cover corresponding to the maximal matching of maximum \(s \rightarrow t\) cut in G.

**Proof.** Let C be the minimum vertex cover corresponding to the maximal matching of maximum \(s \rightarrow t\) cut in G. Also, let B be the cutset for the maximum \(s \rightarrow t\) cut in G and m be the cardinality of the maximal matching corresponding to B. Then from König’s Theorem we can say that |C| = m. We claim that the maximum number of paths that can exist between s and t without any edge overlap is m, given that the paths can only overlap in the edges \(e \in B\).

We prove this lemma by contradiction. Given C, it is trivially true that it is possible to find m paths between s and t through the nodes in C without any edge overlap. Let us assume that \((m + 1)\) such paths are possible between s and t without any edge overlap. To achieve this, we need to have another \(s \rightarrow t\) cutset in G such that the maximal matching corresponding to the cutset has a cardinality equal to \(m + 1\). Let the corresponding minimum vertex cover be C*. Now, we already have assumed that we get the maximum of maximal matching corresponding to the \(s \rightarrow t\) cutsets for C. Hence C must be same as C* or in other words, no such C* exists. Hence \((m + 1)\) paths are not possible between s and t without edge overlap. □

The second way of increasing edge independence is the use of a simple weight-based algorithm to compute the paths between the source and the anchor nodes and then again from the anchor nodes to the destination by gradually incrementing the cost of the edges already accounted for in a path (procedure FindShortest-Paths()). Note, our objective is to maximize node overlap and minimize edge overlap for paths converging at e ∈ C, but not for paths converging at different anchor nodes. According to our routing strategy, the paths converging at an anchor node can leverage the node overlap for partial recovery while the edge independence ensures that each description will be independently affected by wireless link errors.

It may be noted that, our proposed routing and forwarding schemes are agnostic to the networking technologies involved. But they can be adapted to any standard protocol, e.g., IP, with suitable engineering exercise. For example, the packets can be marked in the transport header for merging at certain points. To this end, the routing can have distributed control, as in loose source routing. For dedicated networks, the intermediate nodes can be designed in such a way that they inherently support our proposed store-process-forward mechanism. For the existing shared networks, an overlay can be built for the existing routers to implement the store-process-forward functionalities.

5. Results

In this section, we present results that evaluate our coding and routing scheme. We simulated the experiment environment in MATLAB. The network topology used is similar to that shown in Fig. 4. However, the number of nodes in the mesh was varied. In the experiments evaluating the coding–decoding scheme, unless otherwise mentioned, an equal mix of good and bad links was taken. This was to show the benefit of merging when at least a few descriptions are corrupted. The mix of good and bad links was varied probabilistically. The PER for good links was chosen between 0.1% and 1% and those of bad links between 10% and 20%. The input image was 512 × 512 pixel black-and-white Lena JPEG file.

5.1. PSNR gain due to coding and decoding

In this experiment, three different scenarios were considered. In the first scenario no intermediate recovery was performed. The second scenario employed a naive intermediate recovery technique where link error information was not used and interpolation was based on equal weights to all neighbors. In the third scenario we applied the cross-layer information, wherein no recovery was attempted on the less or non-corrupted descriptions and neighboring PER aware weights were assigned for interpolating a highly corrupted description.

Figure 6 shows the merit of cross-layer decoding in terms of PSNR gain of about 5 dB with respect to no partial recovery. The gain remains nearly unchanged with the increase in number of descriptions.

To show the impact of partial recovery in large networks we considered four chosen descriptions merging at more than one

\(^3\) We assume standard definitions of \(s \rightarrow t\) cut, cutset, maximum cut, maximal matching and a vertex separator, Menger’s Theorem, König’s Theorem.
intermediate recovery stages, where each stage corresponds to a set of intermediate merging points. Figure 7 shows the gain in PSNR against the number of intermediate recovery stages. The first stage gives a gain of about 6 dB, however it does not grow significantly with increase in the number of stages. As larger number of pixels are corrupted, probability of finding good pixels for recovery in the next hop is less. For the recovery case, as the hops are increased, and more pixels are in error, the recovery fails to be as effective. Therefore, with increasing number of hops, the gain in PSNR starts reducing. This observation is important in our routing scheme where we ensure that at least one intermediate merging point is guaranteed but not necessarily strive for multi-stage intermediate merging points. Any further merging points by virtue of route construction can add to a limited gain.

5.2. Evaluating the coding parameters

Understanding the correct choice of the number of descriptions to merge at an intermediate node is important because it determines the number of descriptions to be created at the source. As explained in Section 2, two streams at a merging point makes available two out of four neighboring pixels to aid recovery, whereas four or more streams merging gives three neighboring pixels, thereby improving the chance of error recovery. On the extreme, if all streams merge, then all four neighboring pixels are available. In this experiment, we observe the benefit of having additional neighboring pixels while doing recovery. When two neighboring pixels are available where two streams are merging, we achieved a PSNR of 22.67 dB. The overall PSNR was 22.89 dB when three neighboring pixels were available, and if all four neighbors are present PSNR was 22.91 dB. This validates that with increasing number of pixels it is possible to gain PSNR although it may not be very significant.

In generating multiple descriptions, the parameter shift size $c$ determines the effectiveness of our coding scheme. We took 16 descriptions, and changed the shift value from 1 to 8, and recorded the corresponding PSNRs. As depicted in Fig. 8, we show that the best PSNR is achieved when $c$ is half the number of descriptions, confirming our observation in Section 3.

5.3. Evaluating the routing strategy

We considered a mesh network with symmetric grid structure. Figure 4 shows such a $4 \times 4$ network with 16 nodes. As per our routing algorithm, the set of intermediate merging points is $\{4,7,10,13\}$ for source $s$ at 1 and destination $t$ at 16. The routing algorithm then computes the end-to-end paths between $s$ and $t$. To evaluate our proposed algorithm with respect to our objectives we defined a performance index $PI$, which is a weighted measure of number of intermediate nodes that have 2, 3, and 4 or more path overlaps. Based on our considered recovery filter, we have noted that when two or three descriptions merge, only two neighboring pixels on average can be used for recovery, whereas with four or more descriptions merging three neighboring pixels are available. Accordingly, $PI$ is defined as:

$$PI = \frac{2\eta_2 + 2\eta_3 + 3\eta_4}{7},$$

where $\eta_2$, $\eta_3$, and $\eta_4$ are the nodes with 2, 3, and 4 or more merging of paths.

We compared the performance of our routing strategy with a naive routing algorithm, where the end-to-end shortest paths between $s$ and $t$ are considered in group of 4 similarly as in our proposed algorithm. The paths within each group are discovered with minimal edge overlap following the same weight-based biasing of shortest path algorithm. Comparative performance results for grid networks of various dimensions are shown in Figs. 9 and 10. A significant gain in terms of $PI$ is observed with
As stated in Sections 1 and 2, our work has two distinct research components (i) image coding and decoding and (ii) multipath routing and forwarding in a mesh network. These areas have been researched extensively, albeit individually, and reported copiously in the literature. Below, we highlight the related works that attempted to exploit the benefits of image coding and routing diversity.

In terms of image coding using MDC, there have been several techniques, namely, scalar quantization based MDC (MDSQ), forward error correction (FEC) based MDC, multiple description transform coding (MDTC), and spatial interleaving based MDC. MDTC has the benefits of higher coding efficiency relative to MDSQ (Orchard et al., 1997). Spatial interleaving and MDTC also has the simplicity and widespread compatibility with any source compression algorithm compared to FEC based MDC (Wang et al., 1997).

With respect to cross-layer performance, MDC has garnered significant attention from the researchers for image/video transmission over unpredictable paths due to network congestion as well as wireless link errors (see, e.g., Alasti et al., 2001; Goyal, 2001; Kim et al., 2003; Lee and Radha, 2005). Several newer technical approaches to MDC exist such as the transform technique (e.g., Servetto et al., 2000; Wang et al., 2001), layered coding (e.g., Taal et al., 2007), spatial interleaving (e.g., Gogate et al., 2002; Zheng and Chau, 2005), etc. Depending on the source coding pattern, there have also been several image recovery techniques that exploit the correlation of image blocks resulting from the compression (Wang and Zhu, 1998). Spatial domain interpolation (Ramamurthy and Raychaudhuri, 1995; Hemami and Meng, 1995), which is a block based error concealment technique, specially suits the context of spatial interleaving based MDC.

Although cross-layer optimization has been studied extensively in the context of multipath routing (MPR) in wireline and wireless ad hoc networks, broadband routing requirements have led to joint investigation of the coding (MDC) and routing (MPR) aspects (see, e.g., Goyal, 2001; Gogate et al., 2002; Barrenechea et al., 2003; Wang and Wu, 2004; Diaz et al., 2004; Mao et al., 2005; Mao et al., 2006; Kompella et al., 2007; Yi et al., 2009). The use of path diversity along with MDC was proposed in Apostolopoulos et al. (2002), Wong et al. (2002), Setton et al. (2004), and Apostolopoulos and Trott (2004), primarily to cope with dynamic network congestion. Genetic algorithm based suboptimal MPR solution was used in Mao et al. (2006) to solve a computationally hard problem of finding an optimal multipath route. Sequential approach to k-shortest path search based multipath route construction was proposed in Yi et al. (2009), where a cost metric based routing scheme was introduced to reduce the link overlap. Recently, asymmetric MD coded video transmission over multihop wireless network was studied in Zhou et al. (2009), where the objective was optimal rate allocation to different MDC users to minimize packet losses in different links due to congestion and unreliability. On the optimal video transmission over multi-channel multi-radio multi-hop wireless networks, Zhou et al. (2010) considered distributed packet scheduling to minimize the distortion and achieve fairness among the users. This scheduling policy takes care of optimal channel assignment, rate allocation, and routing over multiple paths.

In contrast to the prior works, in our work, we aimed at generating optimum number of symmetric multiple descriptions on an already transformed (compressed) image depending on the end-to-end routing information, and looked for a suitable spatial interleaving pattern that can be leveraged for intermediate recovery, resulting in an increased error correction. A simple interpolation technique based on four neighboring blocks, as presented in Hemami and Meng (1995), is used for a lost or corrupted block recovery. The routing strategy, on the other hand, focused on optimum number of node overlaps (merging points) and possibly maximize the PSNR gain in the face of random link errors.

It may be noted that, the intermediate recovery of MDC packets over multipath routes is different from network coding concept (Ahlswede et al., 2000) that work in collaboration with routing to reduce the network bandwidth overhead (and, as a result, increase network throughput) at the cost of some additional computation at the nodes.

6. Related work

As stated in Sections 1 and 2, our work has two distinct research components (i) image coding and decoding and (ii) multipath routing and forwarding in a mesh network. These areas have been researched extensively, albeit individually, and reported copiously in the literature. Below, we highlight the related works that attempted to exploit the benefits of image coding and routing diversity.

In terms of image coding using MDC, there have been several techniques, namely, scalar quantization based MDC (MDSQ), forward error correction (FEC) based MDC, multiple description transform coding (MDTC), and spatial interleaving based MDC. MDTC has the benefits of higher coding efficiency relative to MDSQ (Orchard et al., 1997). Spatial interleaving and MDTC also has the simplicity and widespread compatibility with any source compression algorithm compared to FEC based MDC (Wang et al., 1997).
coding (MDC), where a frame is split into multiple smaller descriptions, is useful in guarding a frame against channel errors during transmission. Usually, in MDC a frame is split into two descriptions, and the two descriptions are sent over two different paths, thereby reducing not only the error, but also the end-to-end delay. In this work, we explored the feasibility of splitting an image frame into potentially large number of descriptions and exploited path diversity for intermediate recovery in multihop wireless networks. In order to split the frame into multiple descriptions, we designed a method of assigning pixels to different descriptions such that, the spatial information in the image is well spread out among the different descriptions. Our proposed routing strategy utilizes path diversity in a way that all correlated descriptions are not corrupted simultaneously, and at the same time it also ensures merging of multiple descriptions at intermediate nodes for partial recovery, before reaching the destination. For error recovery at the merging points, we also made use of the spatial correlation component present in an image frame. We adopted a simple $2 \times 2$ filter with our proposed link error aware weight assignments to interpolate a lost or corrupted pixel from its spatial neighbors. The routing scheme does not sacrifice end-to-end delay by using a large number of merging points, which ensures that the descriptions can be sent over edge-disjoint paths simultaneously.

Our simulation experiments in this work have focused mainly on regular mesh networks and the performance gains through proper coding, forwarding, and decoding. As a future work, we will study the routing performance individually as well as the proposed strategies jointly on random networks. Additionally, the performance evaluation exercise to evaluate the network computing requirements is by itself a rigorous exercise and mandates a separate detailed work to understand the challenges of implementing the proposed mechanism with the existing networking technologies and whether it makes sense to have a dedicated network or not. This analysis is out of the scope of this paper and we plan to work on this performance evaluation exercise in our future work.

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