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Use of $k-\epsilon-\gamma$ Model to Predict Intermittency in Turbulent Boundary-Layers

The intermittency profile in the turbulent flat-plate zero pressure-gradient boundary-layer and a thick axisymmetric boundary-layer has been computed using the Reynolds-averaged $k-\epsilon-\gamma$ model, where k denotes turbulent kinetic energy, ϵ its rate of dissipation, and γ intermittency. The Reynolds-averaged model is simpler compared to the conditional model used in the literature. The dissipation equation of the Reynolds-averaged model is modified to account for the effect of entrainment. It has been shown that the model correctly predicts the observed intermittency of the flows.
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1 Introduction

Turbulent boundary-layers are intermittent close to the free-stream where the flow alternates between turbulent and irrotational states. Intermittency at a point is defined as the fraction of time flow is turbulent at that point. Characteristics of the outer intermittent region are important for understanding the entrainment process, in which the outer irrotational fluid becomes turbulent by acquiring vorticity. This process is governed mainly by large eddies. The interface separating turbulent and nonturbulent zones is influenced by eddies of all sizes and is sharp, irregular and changes rapidly. The prediction of intermittency (γ) of wall-bounded flows has not received much attention as intermittency is not as important as the skin friction coefficient and drag which are governed mainly by the near wall dynamics.

Eddy-viscosity based $k-\epsilon$ model is widely used to compute wall-bounded and free-shear flows. Byggstoyl and Kollmann [1] proposed a conditional $k-\epsilon-\gamma$ model in which momentum equations for the turbulent and irrotational zones and transport equations for turbulent kinetic energy and its dissipation are solved, with a contribution of the turbulent-irrotational interface to these equations. They solved the intermittency transport equation based on the turbulent zone quantities. Subsequently, they extended their model to the second moment closure (Byggstoyl and Kollmann [2]). The second-moment conditional model is difficult to implement for wall-bounded flows as near wall modifications for such models are much more complex compared to the eddy-viscosity based models.

Recently, Cho and Chung [3] proposed the $k-\epsilon-\gamma$ model based on the Reynolds-averaged quantities, which is simpler compared to the conditional $k-\epsilon-\gamma$ model. The model of Cho and Chung [3] is superior to the conditional $k-\epsilon-\gamma$ and second moment models in predicting the intermittency of free-shear flows. They showed that the effect of entrainment on the rate of dissipation of turbulent kinetic energy considered in their model removes the weaknesses of the standard $k-\epsilon$ model in predicting different nonbuoyant free-shear flows (axisymmetric jet, plane far wake, and plane mixing-layer). The standard $k-\epsilon$ model overpredicts the growth rate and turbulent quantities of the axisymmetric jet and underpredicts that of the plane far wake and the plane mixing-layer. The $k-\epsilon-\gamma$ of Cho and Chung [3] has been extended to the plane momentumless wake (Ahn and Sung [4]), axisymmetric plume (Dewan et al. [5]) and plane plume (Kalita et al. [6]). Recently, Wang and Derksen [7] have used the $k-\epsilon-\gamma$ model to investigate developing turbulent flow in a pipe. They have showed that transport equation for intermittency based on the Reynolds-averaged quantities eliminates the need for the wall functions and thus the $k-\epsilon-\gamma$ model can be applied to complicated wall-bounded turbulent flows.

The zero pressure-gradient flat-plate boundary layer is the simplest test case for the model. Measurements in an axisymmetric boundary-layer flow on the outer surface of a circular cylinder with boundary-layer thickness greater than the cylinder radius a ($\delta/a>1$; hence termed thick) show that the intermittency is larger compared to the flat-plate boundary-layer. Thick axisymmetric boundary-layer flow is relevant to several engineering applications (Dewan and Arakeri [8]) and is a good test case for the model.

The objective of this paper is to study the Reynolds-averaged $k-\epsilon-\gamma$ model for the flat plate and thick axisymmetric boundary-layers. Section 2 provides the details of the turbulence model and the numerical method used. The modeling of the intermittency equation and the effect of intermittency on mean and turbulent quantities has also been described in Section 2. Section 3 compares the predicted mean and turbulent quantities with measurements for the two flows.

2 Governing Equations and Turbulence Model

The following equations in the cylindrical coordinate system are solved. The boundary-layer assumptions are assumed to be valid. The computations for the flat plate boundary-layer are made in the limiting case with $r/a\rightarrow 0$, where a is the cylinder radius.

Continuity:

$$\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}(rv)=0 \tag{1}$$

Streamwise Momentum Equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left[[r(\nu+\nu_t)]\frac{\partial u}{\partial r}\right] \tag{2}$$

u and v are the mean velocities in the streamwise (x) and radial (r) directions, respectively.

2.1 $k-\epsilon-\gamma$ Model. In the present work we have used the following eddy viscosity relation proposed by Cho and Chung [3] to account for the contribution of the outer irrotational fluid

$$\nu_t=c_\mu\left[1+c_{\mu g}\frac{k^3}{\epsilon^2}\frac{1-\gamma}{\gamma^3}\frac{\partial\gamma}{\partial r}\frac{\partial\gamma}{\partial r}\right]\frac{k^2}{\epsilon} \tag{3}$$

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where, γ denotes intermittency and the model constant $c_{\mu g} = 0.10$. In the limit of fully turbulent flow ($\gamma = 1$) the above relation reduces to the standard eddy-viscosity relation

$$\nu_t = c_\mu \frac{k^2}{\epsilon} \tag{4}$$

where, the standard $k-\epsilon$ model constant $c_\mu = 0.09$. Intermittency (γ) profile, turbulent kinetic energy (k), and its dissipation (ϵ) are obtained from the following transport equations.

Turbulent Kinetic Energy:

$$u\frac{\partial k}{\partial x} + v\frac{\partial k}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left[\left[r\left(\nu + \frac{\nu_t}{\sigma_k}\right)\right]\frac{\partial k}{\partial r}\right] + P_k - \epsilon \tag{5}$$

Rate of Turbulent Kinetic Energy Dissipation:

$$u\frac{\partial \epsilon}{\partial x} + v\frac{\partial \epsilon}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left[\left[r\left(\nu + \frac{\nu_t}{\sigma_\epsilon}\right)\right]\frac{\partial \epsilon}{\partial r}\right] + c_{\epsilon 1}\frac{\epsilon}{k}P_k + c_{\epsilon 4}\Gamma\frac{\epsilon^2}{k} - c_{\epsilon 2}\frac{\epsilon^2}{k} \tag{6}$$

where, $P_k = \nu_t S^2$ is the production of turbulent kinetic energy with mean strain rate $S = [\partial u/\partial r + \partial v/\partial x]$. For the near wall region, the low-Re $k-\epsilon$ model of Chien was used, which is quite accurate in predicting the near wall quantities for the flat plate boundary-layer (Patel et al. [9]). The predicted characteristics of the outer region are independent of the near-wall modifications as in the outer region all the low-Re $k-\epsilon$ models reduce to the standard $k-\epsilon$ model. Therefore, the near wall version used in the model is not important. $c_{\epsilon 1} = 1.35$, $c_{\epsilon 2} = 1.88$, $\sigma_k = 1.0$ and $\sigma_\epsilon = 1.3$ are the constants of Chien's low-Reynolds number $k-\epsilon$ model (Patel et al. [9]). In the present work we have added the third term on the RHS of the dissipation equation and its effect is discussed in the next section. It may be noted that in Section 3.1 predictions using the low-Re $k-\epsilon$ model of Chien (termed as the $k-\epsilon$ model in the present paper) employ the standard eddy viscosity relation (4).

2.2 Intermittency. Libby [10] was the first to propose a model for a guessed transport equation for the intermittency. Dopazo [11] derived an exact transport equation for intermittency by multiplying the instantaneous continuity equation by the intermittency indicator function and taking the ensemble average. The intermittency indicator function is used to distinguish between the two zones and is equal to one in the turbulent zone and zero in the nonturbulent zone. The exact intermittency equation involves terms accounting for the transport of intermittency due to the relative motion of turbulent and nonturbulent zones, and production terms. The following modeled equation for intermittency is solved

$$u\frac{\partial \gamma}{\partial x} + v\frac{\partial \gamma}{\partial r} = D_g + S_g \tag{7}$$

D_g represents the transport of γ due to the mean velocity difference between the turbulent and irrotational fluids and is effective only in the intermittent region. The following diffusion model for D_g is used (Cho and Chung [3]): $D_g = (1/r)(\partial/\partial r)[r(1-\gamma)(\nu_t/\sigma_g)(\partial\gamma/\partial r)]$, with the model constant $\sigma_g = 1.0$.

S_g represents the conversion rate of the outer irrotational fluid into the turbulent fluid and involves the geometry of the interface. The following model for S_g is used, with the model constants $c_{g1} = 1.6$, $c_{g2} = 0.15$, $c_{g3} = 0.16$ (Dewan et al. [5])

$$S_g = c_{g1}\gamma(1-\gamma)\frac{P_k}{k} + c_{g2}\frac{k^2}{\epsilon}\frac{\partial\gamma}{\partial x_j}\frac{\partial\gamma}{\partial x_j} - c_{g3}\gamma(1-\gamma)\frac{\epsilon}{k}\Gamma \tag{8}$$

The last term in the model for S_g accounts for the effect of entrainment. This term, however, is negligible for most flows. Here $\Gamma = (k^{5/2}/\epsilon^2)U_i/(U_k U_k)^{1/2}(\partial U_i)/(\partial x_j)(\partial\gamma/\partial x_j)$ is the intermittency interaction invariant and is a measure of the change in intermittency due to the entrainment (Cho and Chung [3]). In the present work with boundary-layer assumptions Γ is approximately equal to $k^{5/2}/\epsilon^2 \partial u/\partial r \partial\gamma/\partial r$. The magnitude of Γ depends on the rate of entrainment. If the entrainment is by the inward movement of the outer irrotational fluid (as in a jet or in a plume) then γ at a fixed location decreases and vice-versa. A reduction in γ means reduction in turbulent length scale l or an increase in ϵ as $l \simeq k^{3/2}/\epsilon$.

Cho and Chung [3] proposed this model by modifying the model of Byggstoyl and Kollmann [2] which was used with the conditional $k-\epsilon$ and Reynolds stress transport models. Dewan et al. [5] modified the model of Cho and Chung [3] to account for the effect of buoyancy on intermittency and studied intermittency of the axisymmetric plume. Recently, Kalita et al. [6] have applied this model to the plane plume. Note that the intermittency equation does not require any special near wall treatment. Cho and Chung [3] also modified the dissipation equation (6) by adding the term $c_{\epsilon 4}\Gamma\epsilon^2/k$ with the model constant $c_{\epsilon 4} = 0.1$. This term accounts for the effect of entrainment and is significant. Its inclusion in the dissipation equation removes the limitations of the standard $k-\epsilon$ model in predicting different free-shear flows (described in Section 1).

2.3 Numerical Method. The governing equations were solved using the control volume method (Patankar [12]). Power-law scheme (Patankar [12]) in the cross-stream direction and upwind scheme (Anderson et al. [13]) in the stream-wise direction were used. Grid independence was checked and achieved. For one typical case with $\mathrm{Re}_a = 736$ increasing the number of radial grids from 100–150 resulted in the changes in C_f and H at $\mathrm{Re}_{\theta,ax} = 4350$ by the two models within 0.3 percent. The computations reported in this paper are using 100 radial grid points. To resolve sharp gradients close to the wall about 30 grid points were located within $y^+ = 50$. An axial step size equal to the smaller of five sublayer thickness and 25 percent of momentum thickness was used (Patel et al. [9]).

At the wall u and k were set to be zero, ϵ was computed based on Chien's low-Re $k-\epsilon$ model and the intermittency was set to unity at the wall (Wang and Derksen [7]). At the free-stream the variables k, ϵ, and γ were set to zero. Computations were started sufficiently upstream of the region of interest. At the starting point a small trigger in the values of k and ϵ was given to initiate turbulent flow. Computations were made to study the dependence of results on the free-stream turbulence level defined by $T = 1/U_\infty[2k/3]^{1/2}$ (White [14]) by considering three values of T equal to 0.5 percent, 1 percent, and 2 percent, where U_∞ is the free-stream velocity. The results reported in the present paper are independent of values of k and ϵ at the starting point and the value of free-stream turbulence level. The code was validated with the results reported by Patel et al. [9] for the flat plate boundary-layer using the low-Re $k-\epsilon$ model and for the axisymmetric jet by Cho and Chung [3] using the Reynolds-averaged $k-\epsilon-\gamma$ model. The predicted C_f, H, mean and turbulent quantities were within 2 percent of those reported in the above references (Dewan [15]).

3 Results and Discussion

3.1 Flat Plate Boundary-Layer. Figures 1 and 2 compare the predicted intermittency and turbulent kinetic energy profiles at $\mathrm{Re}_\theta = 2600$ with measurements by Klebanoff [16] for the flat plate boundary-layer with zero pressure gradient. The prediction of intermittency by the conditional $k-\epsilon-\gamma$ model of Byggstoyl and Kollmann [1] and the prediction of turbulent kinetic energy profile by low-Re $k-\epsilon$ model of Chien (hereafter termed as the $k-\epsilon$ model) have also been shown in Figs. 1 and 2, respectively. The prediction of intermittency by the Reynolds-averaged $k-\epsilon-\gamma$ model is superior to that by the conditional $k-\epsilon-\gamma$ model (Fig. 1). While using the Reynolds-averaged $k-\epsilon-\gamma$ model, the Eq. (6) for dissipation has been modified to account for the effect of entrainment, by adding the term containing intermittency interaction invariant Γ. The effect of entrainment on dissipation depends on the direction of motion of the interface and its magnitude de-

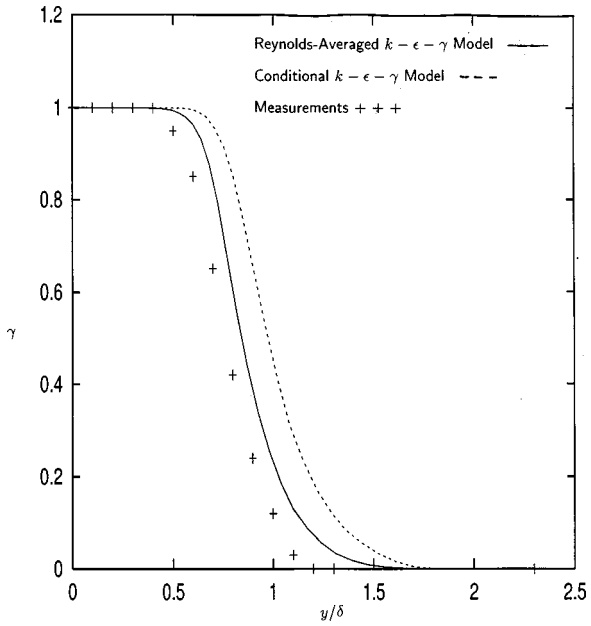


Fig. 1 Predicted intermittency profiles for the flat plate boundary layer at Re_θ=2600 compared with the measurements

pends on the rate of entrainment. For boundary-layer flow the effect is to slightly increase dissipation (or decrease k) in the outer intermittent region (Fig. 2). The overall deviation between predictions by the two models and measurements in the outer region (Fig. 2) is due to the different values of the model constants used in Chien's low-Re $k-\epsilon$ model (Patel et al. [9]). Table 1 shows that the values of the skin friction coefficient and shape factor predicted by the two models are almost the same.

There is a debate in the literature on the effect of the outer irrotational fluid to the eddy viscosity. Based on the DNS data for the flat plate boundary-layer, Cazalbou and Bradshaw [17] have shown that close to the free-stream the standard eddy-viscosity

Table 1 Predicted skin friction coefficient and shape factor for flat plate boundary-layer at Re_θ=2600 compared with the measurements

Quantity	Measurement	Low-Re $k-\epsilon-\gamma$ model	Low-Re $k-\epsilon$ model
$C_f\times10^3$	3.42	3.41	3.43
H	-	1.34	1.35

model fails as here the value of the constant c_μ in the eddy-viscosity relation becomes three to four times its original value of 0.09. They suggested that this is probably due to the intermittent nature of the flow close to the free stream in wall-bounded and free-shear flows. It may be noted that the predictions of the eddy-viscosity by the relation (3) are consistent with the observations of Cazalbou and Bradshaw [17] and show a large increase in the value of the constant c_μ close to the free-stream.

3.2 Thick Axisymmetric Boundary-Layer. For this flow the computations have been compared with measurements for the following three cases: (a) Re_a=4800 and δ/a=4.59; (b) Re_a=3200 and δ/a=7.57; and (c) Re_a=736 and $Re_{\theta,ax}$=4350 ($\delta/a\simeq40$). Transverse curvature effects in the outer region become more important as the value of δ/a increases. Figure 3 compares Reynolds shear stress profiles for the case (a). Figures 4 and 5 compare intermittency and Reynolds shear-stress profiles for the case (b). The measurements of intermittency profile by Lueptow and Haritonidis [18] and of Reynolds shear-stress profile by Lueptow et al. [19] have also been shown in Figures 3, 4, and 5 for comparison. For the case (b) the predicted intermittency profile by the model is one over larger boundary layer thickness compared to the flat plate boundary-layer and the agreement with the measurements is good (Figs. 1 and 4). Here larger γ is observed because of less constraint on the motion of eddies compared to the flat-plate boundary-layer. With an increase in δ/a the Reynolds shear-stress in the outer region decays faster compared to the flat plate boundary-layer as turbulence generated in the near wall region sustains larger area compared to flat plate boundary-layer (Fig. 3). The decay of Reynolds shear-stress in the outer region is larger

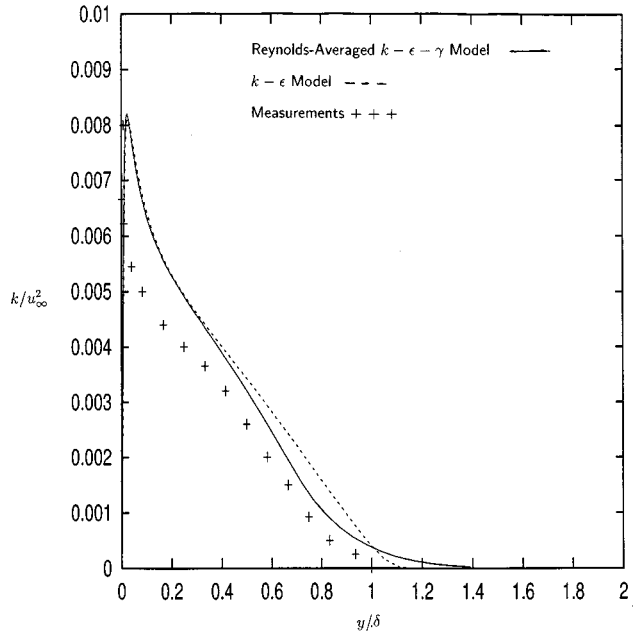


Fig. 2 Predicted turbulent kinetic energy profiles for the flat plate boundary-layer at Re_θ=2600 compared with the measurements

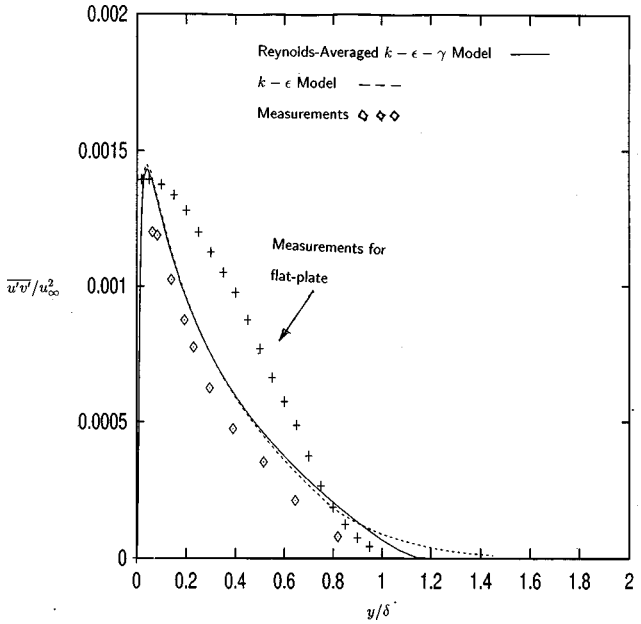


Fig. 3 Predicted Reynolds shear-stress profiles for thick axisymmetric boundary-layer (Re_a=4800, δ/a=4.59) compared with the measurements

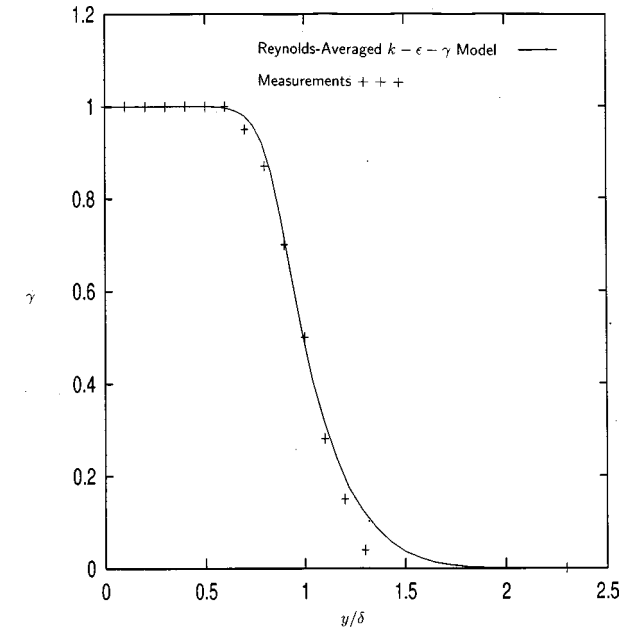


Fig. 4 Predicted intermittency profile for thick axisymmetric boundary-layer (Re_a=3200, δ/a=7.57) compared with the measurements

for the case (b) compared to the case (a) because of larger value of the boundary-layer thickness in the latter. The models correctly predict this behavior (Figs. 3 and 5).

Tables 2, 3, and 4 compare the predicted skin friction coefficient and shape factor with the measurements by Lueptow et al. [19] for cases (a) and (b) and Willmarth et al. [20] for case (c) of thick axisymmetric boundary-layer. Here $Re_a = U_\infty a/\upsilon$ and $Re_{\theta,ax} = U_\infty \theta_{ax}/\upsilon$, where U_∞ is the free-stream velocity, a is the cylinder radius, θ_{ax} is the momentum thickness for axisymmetric flows (Willmarth et al. [20]), and υ is the kinematic viscosity. Underprediction in the maximum value of Reynolds shear stress profiles

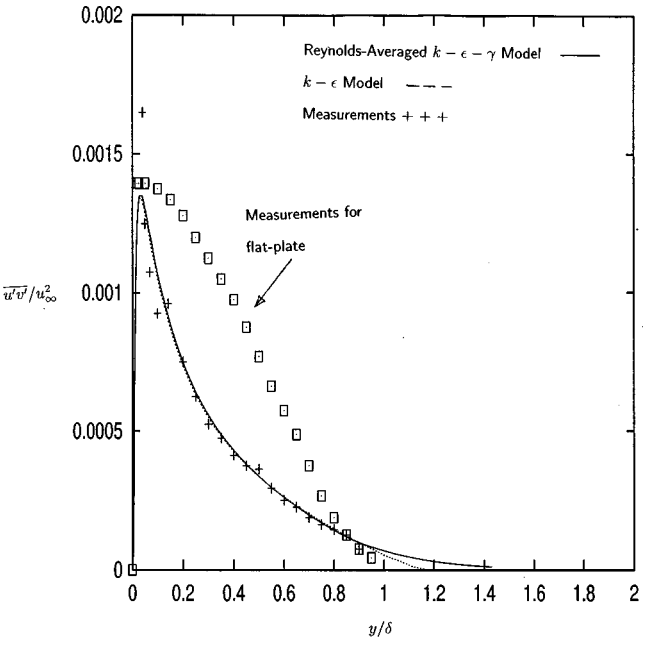


Fig. 5 Predicted Reynolds shear-stress profiles for thick axisymmetric boundary-layer (Re_a=3200, δ/a=7.57) compared with the measurements

Table 2 Predicted skin friction coefficient and shape factor for thick axisymmetric boundary-layer at Re_a=4800 and δ/a=4.59 compared with the measurements

Quantity	Measurement	Low-Re $k-\epsilon-\gamma$ model	Low-Re $k-\epsilon$ model
$C_f \times 10^3$	4.61	3.84	3.80
H	-	1.16	1.16

Table 3 Predicted skin friction coefficient and shape factor for thick axisymmetric boundary-layer at Re_a=3200 and δ/a=7.57 compared with the measurements

Quantity	Measurement	Low-Re $k-\epsilon-\gamma$ model	Low-Re $k-\epsilon$ model
$C_f \times 10^3$	4.80	3.88	3.85
H	-	1.113	1.116

Table 4 Predicted skin friction coefficient and shape factor for thick axisymmetric boundary-layer at Re_a=736 and $Re_{\theta,ax}$ =4350($\delta/a \simeq 40$) compared with the measurements

Quantity	Measurement	Low-Re $k-\epsilon-\gamma$ model	Low-Re $k-\epsilon$ model
$C_f \times 10^3$	7.84	4.74	4.72
H	1.115	1.045	1.047

by the two models for the case (b) of thick axisymmetric boundary-layer (Fig. 5) leads to underprediction of the skin friction coefficient (Table 3). Results for other cases also show similar trends. The large underprediction in C_f for thick axisymmetric boundary layers by different low-Re $k-\epsilon$ models has been discussed by Dewan and Arakeri [8]. They have shown that different low-Reynolds number modifications to the standard $k-\epsilon$ model are unsatisfactory in predicting the near-wall characteristics of such flows. They have shown that a small modification to the zero equation model of Cebeci [21,22] by replacing the standard expression of displacement thickness for planar flows by that for axisymmetric flows leads to superior predictions compared to that using different low-Re $k-\epsilon$ models.

Tables 2, 3, and 4 show that the $k-\epsilon-\gamma$ model with the effect of intermittency on dissipation does not lead to any improvement in the predicted skin friction coefficient compared to the low-Re $k-\epsilon$ model of Chien. The predicted mean and turbulent quantities by the two models are almost same. This is because the effect of entrainment on dissipation accounted for in the Reynolds averaged $k-\epsilon-\gamma$ model marginally affects the flow in the outer region only. In contrast, for most free-shear flows the Reynolds-averaged $k-\epsilon-\gamma$ significantly improves predictions compared to the standard $k-\epsilon$ model.

The results show that the present Reynolds-averaged $k-\epsilon-\gamma$ model accurately predicts the outer region of wall-bounded flows. The fact that this model is also appropriate for different free-shear flows shows that it could be a universal model for intermittency.

4 Concluding Remarks

We have shown that the $k-\epsilon-\gamma$ model based on the Reynolds averaged quantities is adequate to predict the intermittency of wall-bounded flows. Making near wall modifications to this model as used in any low Reynolds number version of the $k-\epsilon$ model results in a model which can be used to study the near wall and outer characteristics of the turbulent flat plate boundary-layer and thick axisymmetric boundary-layers. This model is appropriate to study intermittency of different free-shear flows and removes the

weaknesses of the standard $k-\epsilon$ model for free-shear flows. Because of its simplicity the Reynolds-averaged $k-\epsilon-\gamma$ model could be extended, with small modifications, to the axisymmetric plume and plane plume, buoyancy driven free-shear flows. The flow in a plume is more complex than in nonbuoyant free-shear flows with buoyancy affecting mean and turbulent quantities.

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