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PREDICTION OF TURBULENT PLANE JET IN CROSSFLOW

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In this article numerical predictions of turbulent plane jets discharged normal to a weak or moderate cross stream are presented. The Reynolds-averaged Navier–Stokes equations with the standard k–ε turbulence model have been used to formulate the flow problem. The governing equations that are elliptic in nature are solved using the finite volume method. The predictions are presented to illustrate the flow pattern involved and to assess the performance of the standard k–ε model by comparison with available experimental data for three different jet to cross stream velocity ratios (six, nine, and ten) and the agreement is found to be satisfactory.

INTRODUCTION

Turbulent jets in a crossflow have been used in many engineering fields. Among various configurations of jets in a crossflow, turbulent plane jets injected normally into a uniform cross stream have numerous applications. Film cooling, effluent discharged into the atmosphere and into water bodies, and industrial mixing are a few examples of such applications.

The problem of a turbulent plane jet discharged through a slot into a crossflow has received considerably less attention in comparison to the round or rectangular jet in a crossflow. A turbulent plane jet in a crossflow differs from the turbulent round or rectangular jet as in the former the jet stream completely blocks the path of the cross stream in the near field of the jet discharge. According to Demuren [1] the flow field of the plane turbulent jet is nominally three dimensional, and the computation of a two-dimensional flow field is adequate for such a flow configuration.

Carter [2] studied heated turbulent plane jets in a confined crossflow where he measured the temperature trajectories, defined as the locus of local maximum temperature, for three different values of jet to cross stream velocity ratios (R). Ramaprian and Haniu [3, 4] reported their experimental findings of turbulent plane nonbuoyant isothermal and heated jets in a mild crossflow for $R = 6.0$, 9.0, and 10.0. They have measured mean velocity components and their fluctuations together with the mean temperature, temperature fluctuations, and heat fluxes for heated jets. They also checked the two dimensionality of the flow field and concluded that flow can be considered as two dimensional (within 5% of the mean velocity variation).

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NOMENCLATURE

c	model constant
D	nozzle width at the exit
k	turbulent kinetic energy
n	coordinate perpendicular to jet trajectory
p	mean pressure
P_k	production of k
R	jet to cross stream velocity ratio ($= u_j/v_a$)
s	distance along the jet trajectory from the middle of jet slot
u	mean velocity along the direction of the jet at the discharge (x-direction)
U_m	local maximum of the mean velocity
U_{mean}	mean velocity
u'	fluctuating velocity component in x-direction
u'_{rms}	root mean square value of u'
v	mean velocity along cross stream direction (y-direction)
v'	fluctuating velocity component in y-direction
x	coordinate along the direction of the jet at the discharge
y	coordinate along cross stream direction
ε	rate of dissipation of k
ν_t	eddy viscosity
ρ	density
σ_ε	model constant
σ_k	model constant

Subscripts

a	ambient condition
j	jet condition at the discharge
max	maximum value

Ramaprian and Haniu [5] also have reported the measurements of mean and turbulent quantities for the case of buoyant jets for the same configuration. Flacks et al. [6] measured the velocity field in the mixing region of a slot jet injected normal to a confined crossflow for three different jet to cross stream momentum ratios, viz., 3.1, 8.1, and 16.2. They also investigated the two dimensionality of the flow field and concluded that the flow does not exhibit three dimensionality in the region under consideration. McGuirk and Rodi [7] numerically predicted turbulent plane jets in a crossflow of a confined environment. They studied the side discharge into a rectangular channel flow using the depth-averaged version of the $k-\varepsilon$ model [8] and predicted the jet trajectories, stream line pattern, and recirculation eddy size. They presented comparison of their predictions with the experimental observations of Mikhail et al. [9] and Carter [2] and the agreement is reasonably good. Sarkar and Bose [10] predicted characteristics of a two-dimensional turbulent plane jet in a crossflow where a cold jet stream is discharged into a strong cross stream ($R \leq 1.0$) of hot fluid. They compared the performances of five different turbulence models. The models they have tested are low-Reynolds-number versions of the $k-\varepsilon$ model by Lam and Bremhorst [11] and Chien [12], the low Reynolds number version of the $k-\omega$ model of Wilcox [13], and the algebraic eddy viscosity model of Baldwin and Lomax [14] and its modified form. They found that the performance of the Lam–Bremhorst [11] model is relatively good. However, they have not made any comparison with experimental observation for the flow parameters.

The standard $k-\varepsilon$ model [15] has been used extensively to simulate the turbulent round jets in a crossflow. Patankar et al. [16], Catalano et al. [17], Hwang and Chiang [18], He et al. [19], and many others have predicted round turbulent jets in a weak and moderate crossflow ($R \geq 2$) using the standard $k-\varepsilon$ model. Predictions of turbulent plane jet in a weak or moderate crossflow have not been found in the literature. The lack of enough experimental data may be one reason for this. In the current work predictions of turbulent plane jets in a uniform weak and moderate crossflow are presented using the standard $k-\varepsilon$ model. The flow configuration and

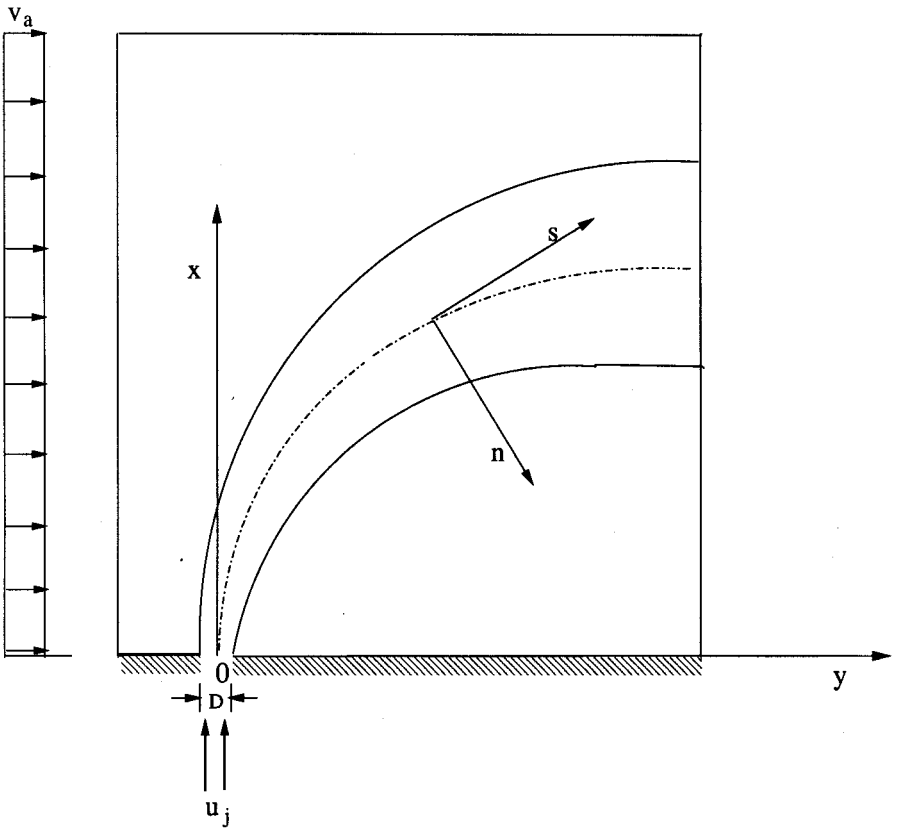


Figure 1. Flow configuration and coordinate system of the problem.

associated coordinates are shown in Figure 1. The predicted results are compared with the available experimental observations [3, 4]. The mathematical formulation and numerical method used are described in the next two sections, followed by results and discussions.

MATHEMATICAL FORMULATION

Governing Equations

We consider a two-dimensional flow field where the jet is discharged normally with a uniform velocity of u_j through a slot of width D into a uniform cross stream of velocity v_a. The flow is assumed to be incompressible and the molecular viscosity is neglected except in the near wall region. Thus the Reynolds-averaged governing equations can be written as

(i) Continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

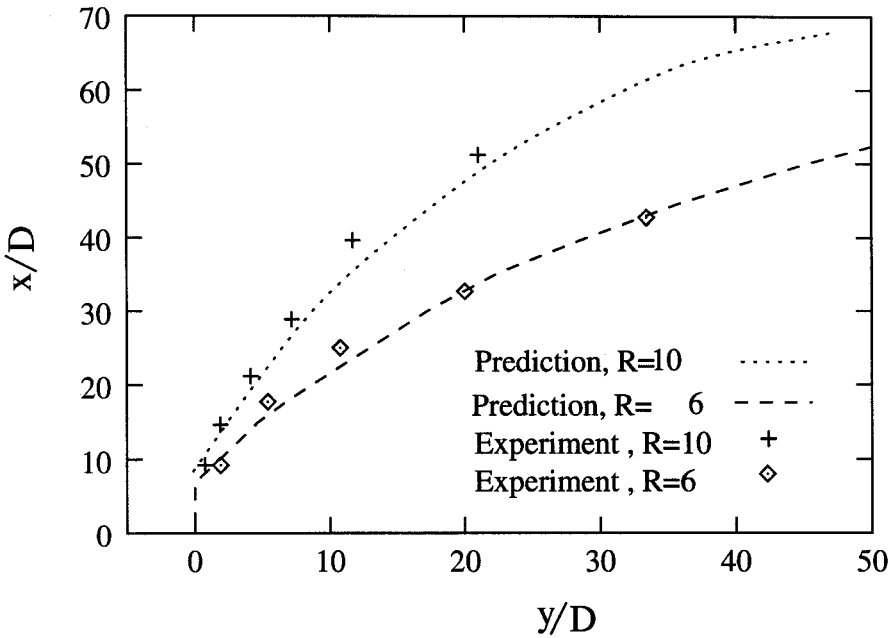


Figure 2. Predicted jet trajectories of turbulent plane jet in a crossflow compared with the measurements for $R = 6$ and 10.

(ii) x-momentum

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}(-\overline{u'^2})+\frac{\partial}{\partial y}(-\overline{u'v'}) \tag{2}$$

(iii) y-momentum

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}(-\overline{u'v'})+\frac{\partial}{\partial y}(-\overline{v'^2}) \tag{3}$$

Here u and v are the mean velocities along the direction of the jet at the discharge (x) and crossflow (y), respectively, and p is the mean pressure. The primed quantities represent fluctuations.

Turbulence Model

We have used the standard $k-\varepsilon$ model [15] and the turbulent stresses are modeled as

$$-\overline{u'v'}=\nu_t\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \tag{4}$$

$$-\overline{u'^2}=2\nu_t\left(\frac{\partial u}{\partial x}\right)-\frac{2}{3}k \tag{5}$$

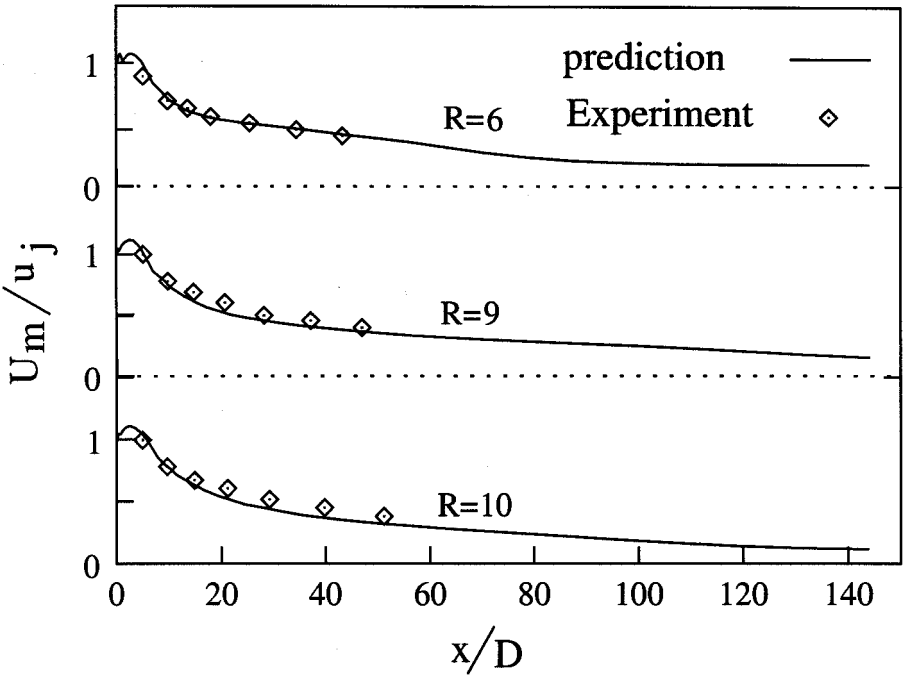


Figure 3. Predicted maximum mean velocity decay of turbulent plane jets in a crossflow compared with the measurements for $R = 6, 9$, and 10.

$$-\overline{v'^2} = 2\nu_t\left(\frac{\partial v}{\partial y}\right) - \frac{2}{3}k \tag{6}$$

where

$$\nu_t = c_\mu \frac{k^2}{\varepsilon} \tag{7}$$

Here ν_t is the eddy viscosity, c_μ is a model constant, k is the turbulent kinetic energy, and ε is the dissipation rate of the turbulent kinetic energy. The transport equations for turbulent kinetic energy and its dissipation are given by

$$u\frac{\partial k}{\partial x} + v\frac{\partial k}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial y}\right) + P_k - \varepsilon \tag{8}$$

$$u\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right) + c_1\frac{\varepsilon}{k}P_k - c_2\frac{\varepsilon^2}{k} \tag{9}$$

where P_k is the rate of production of k and is given by

$$P_k = \nu_t\left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2\right] \tag{10}$$

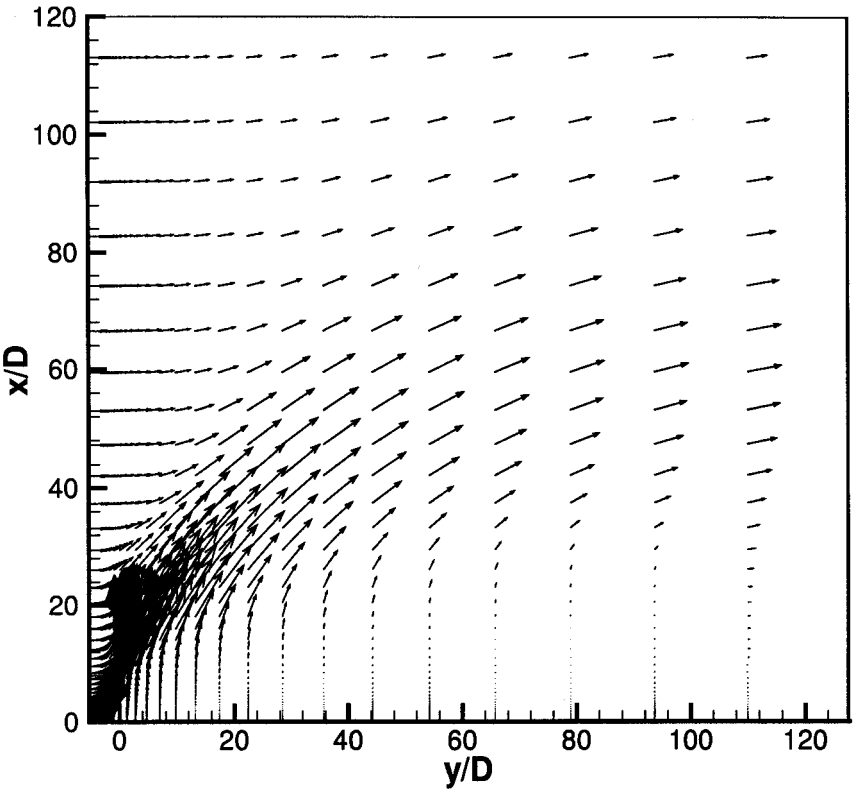


Figure 4. Predicted mean velocity vector plot of a turbulent plane jet in crossflow for $R = 10$.

The values of the model constants are taken as $c_\mu = 0.09$, $c_1 = 1.44$, $c_2 = 1.92$, $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.3$, as suggested by Launder and Spalding [15].

Computational Domain and Boundary Conditions

A rectangular domain is considered for the present computation. In the direction of crossflow, the upstream (left) boundary is considered sufficiently far away from the jet slot, as suggested by Amer et al. [20]. The downstream (right) boundary is placed sufficiently away from the jet slot such that a zero gradient condition for all the variables can be applied [21]. The wall from which the jet emanates constitutes the bottom boundary of the domain, and the top boundary is placed sufficiently away from the bottom so that an initial cross stream velocity condition prevails there [18]. The size of the computational domain depends upon R. For example, for $R = 10$ we consider a domain of $(5D + 120D)$ along the y-direction and $240D$ along x-direction.

Except at the jet discharge, the bottom boundary is considered as a solid wall where the boundary conditions are obtained using the wall function [15]. It is assumed that the universal logarithmic velocity profile prevails in the vicinity of the wall. Turbulent kinetic energy is zero at the wall, and its dissipation rate in the near wall region is assumed to be inversely proportional to the distance from the wall [16]. For the other three boundaries and at the jet origin we have used the conditions as

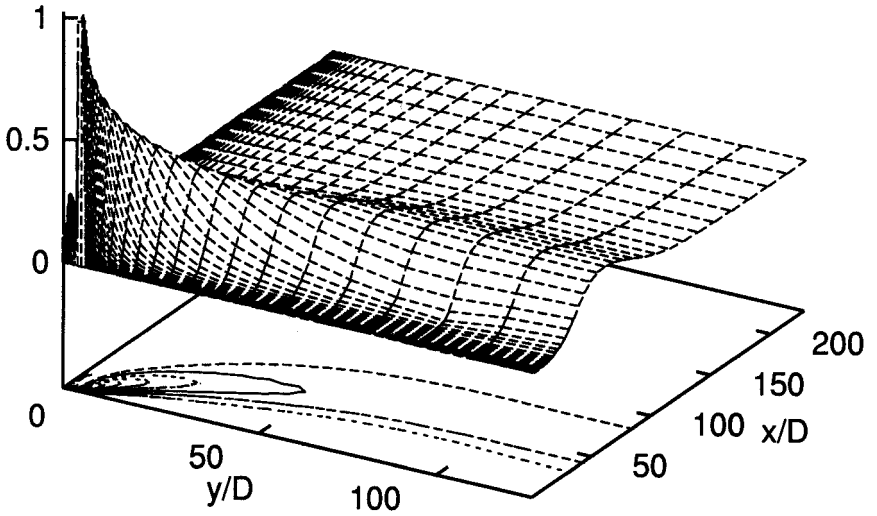


Figure 5. Predicted mean velocity distribution and isocontours of turbulent plane jet in crossflow for $R = 10$.

suggested by Hwang and Chiang [18]. The conditions for the left boundary are $u = 0$, $v = v_a$, $k = 0.04v_a^2$, and $\varepsilon = k^{3/2}/0.06x_{\max}$. The same conditions are applied at the top boundary. At the right boundary normal gradient of all the variables are considered to be zero. At the jet discharge we consider $u = u_j$, $v = 0$, $k_j = 0.001u_j^2$, and $\varepsilon = k^{3/2}/0.5D$. As the velocities at the boundaries are specified, no information about pressure is needed at the boundary [16] and therefore no boundary value for the pressure is prescribed.

NUMERICAL METHOD

In the current study we have used the finite-volume method with nonuniform staggered grids where the velocity components are positioned in the middle of the sides of a control volume, and other variables (p, k, and ε) are positioned in the center of the control volume [22]. The equations involved in this study are elliptic in nature. To solve these equations a computer code is developed based on the SIMPLER algorithm [22]. The convection terms together with the diffusion terms of the transport equations for all the variables are discretized using the power-law scheme [22]. The systems of linear algebraic equations are solved by alternating the direction of the line-by-line iteration method using the tri-diagonal matrix algorithm. At each iteration, the variables u, v, k, and ε are underrelaxed. The values of the underrelaxation parameters are taken as 0.45 for u, k, and ε, and 0.75 for v.

To validate the present computer code, we have predicted the centerline velocity decay of the turbulent confined impinging slot jet flow and compared the results with those of Seyedein et al. [23] and the agreement is found to be good. The difference between the two profiles is less than 4% at any location. For this purpose we have used the wall functions [15] for both the top and the bottom boundaries.

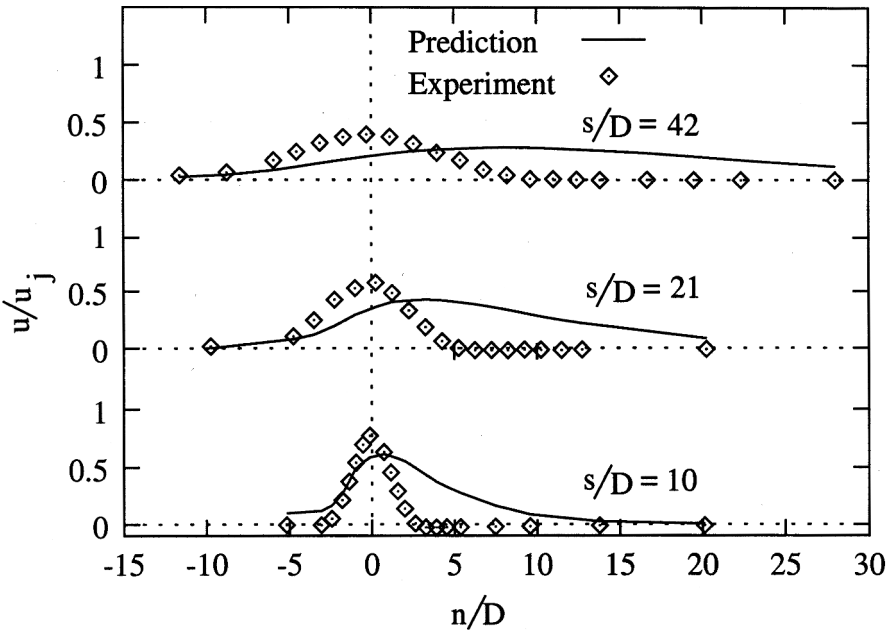


Figure 6. Predicted u/u_j profiles of turbulent plane jet in crossflow at different s/D locations compared with the measurements for $R = 10$.

The grid sensitivity of the present computation is tested using three different sets of grids, viz., 50×30 (50 along the x- and 30 along the y-direction), 60×40, and 70×50. The predicted profiles of the centerline velocity decay have been used to check the grid sensitivity of the computation. From the comparison it is observed that the variations between the predicted results with grid numbers 60×40 and 70×50 are less then 2% and between 50×30 and 60×40 the difference is 3%.

RESULTS AND DISCUSSIONS

There are not many experimental observations for the flow situation considered here. We have found only the observations made by Ramaprian and Haniu [3, 4] and, therefore, present predictions are compared with their observations only.

Jet trajectory, one of the important characteristics of a jet in a crossflow, is defined as the locus of the local maximum velocity. From the analysis of the trajectory, we can obtain a qualitive representation of the flow field and jet deflection. In Figure 2 predicted jet trajectories are compared with the measured data [4] for $R = 6$ and 10. From the comparison it is observed that agreement between predictions and experimental data is fairly good. The figure shows that the lower the value of R, the higher the jet deflection. Due to the mixing of the two streams the local maximum mean velocity (U_m) decreases along the jet trajectory. In Figure 3 the nondimensional maximum velocity decay is compared with the experimental data [3]. The plots show that predicted profiles agree well with experimental observations. The predicted profiles show that in the vicinity of the jet origin ($x/D \approx 3$) slight acceleration of the

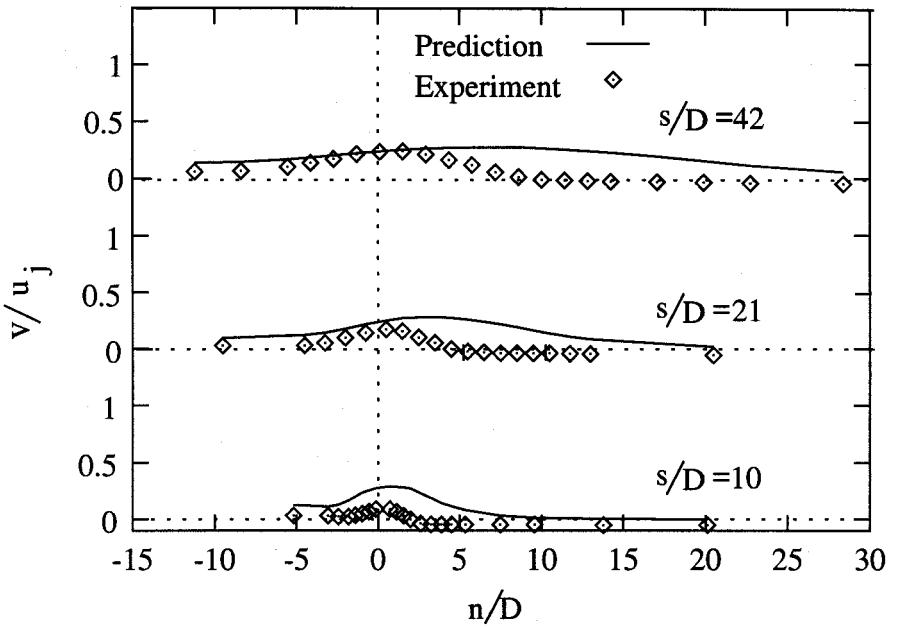


Figure 7. Predicted v/u_j profiles of turbulent plane jet in crossflow at different s/D locations compared with the measurements for $R = 10$.

flow occurs. A similar observation is also reported by Sarkar and Bose [10]. However, no experimental data are available in that region for comparison with the predictions.

The mean velocity vector plot of the flow field is presented in Figure 4 for $R = 10$. From the plot it is observed that the cross stream is completely blocked by the jet stream in the near field and then the jet stream becomes deflected under the influence of the cross stream. The amount of penetration of the jet into the crossflow and deflection of the jet depend upon the value of R. From the present predictions (Figure 2) we can observe that the higher the value of R, the more penetration and less deflection. The mean velocity (U_{mean}) distribution for the whole computational domain is shown in Figure 5 together with its isocontours for $R = 10$. The figure illustrates the details of the flow field generated due to the mutual interaction of the jet and the cross stream. It is observed that the jet width increases along the length of the jet. Predictions for $R = 6$ and 9 also show similar trends (not shown here).

Present predictions of the mean velocity components are compared with the experimental observations [3] at different locations of the flow field for $R = 10$ in Figures 6 and 7. The experimental data are available in the s– n coordinates where s is along the jet trajectory and n is normal to the jet trajectory. The plots of non-dimensional flow parameters against n/D at different values of s/D have been used for the comparison of the present predictions with the experimental observations. Figure 6 shows that agreement between the predicted and the experimental values of the mean velocity in the x-direction is reasonably good. The comparison of profiles of the mean velocity in the direction of crossflow (Figure 7) shows that the predicted values are higher than the experimental data for lower values of s/D. Considering

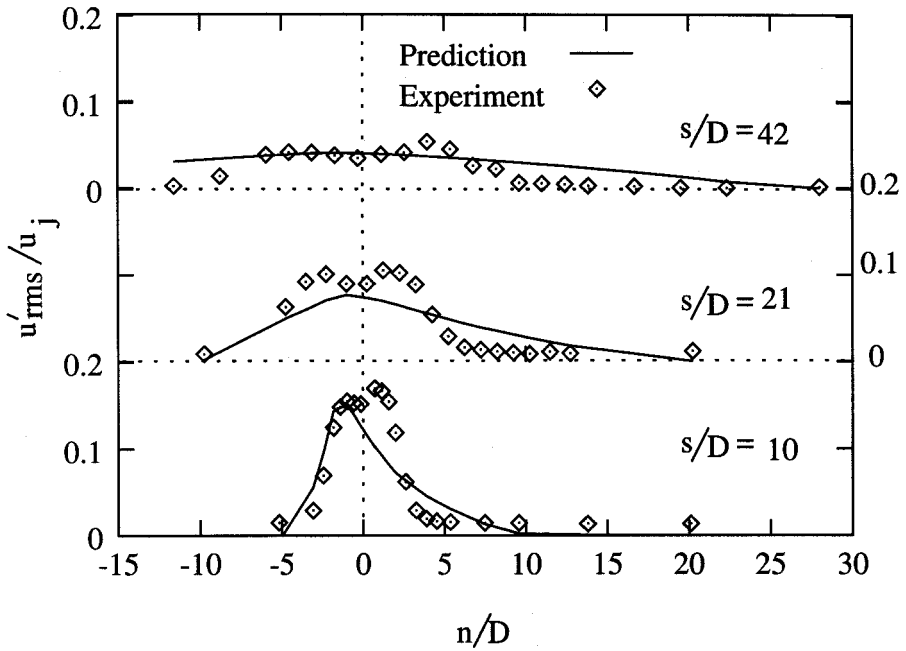


Figure 8. Predicted u'_{rms}/u_j profiles of turbulent plane jet in crossflow at different s/D locations compared with the measurements for $R = 10$.

the possibility of presence of large experimental scatter for the measurement of v for the whole range of measurements along n, as mentioned by Ramaprian and Haniu [3], the present predictions are satisfactory. The plots of the root mean square values of the fluctuating velocity component in the x-direction (Figure 8) show that the agreement between the predictions and the measurements is reasonably good.

CONCLUSIONS

It is shown that the important flow configuration of turbulent plane jets in weak and moderate crossflows can be predicted using the standard $k-\varepsilon$ model with moderate success. The comparison between the present predictions and the experimental data shows good qualitative agreement and reasonable quantitative agreement on most features.

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