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Numerical Study of Three-Dimensional Jets using Point Source Method

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Abstract

Jets emanating from non-axisymmetric sources have been simulated using the point source method, based on Reichardt's hypothesis. The non-axisymmetric shapes that have been studied are rectangular, elliptic, and triangular geometries. In the case of rectangular and elliptic jets, four aspect ratios have been considered. The results obtained for a circular jet have been compared with those obtained from finite difference method, and measurements. The results demonstrate that the point source method is a simple and powerful technique to obtain the gross features of jets.

Notation

AR	Aspect ratio
$b_{0.5}$	Half velocity width
D	Equivalent diameter
B_0	Height of the orifice
L_0	Length of the orifice
U	Velocity
U_c	Centreline velocity
U_0	Velocity at the jet exit
X	Axial distance
$Y_{0.5}$	Half velocity width along Y direction
$Z_{0.5}$	Half velocity width along Z direction

1. Introduction

Jet mixing has important applications in mechanical and aerospace engineering systems. For instance, combustion and propulsive systems require both large scale and fine scale mixing. Hence, there is a sustained effort to enhance jet mixing. In this direction, non-circular jets have proven themselves to be effective passive control mechanisms for mixing enhancement. The flow emanating from a non-circular nozzle is three-dimensional, turbulent, and poses considerable difficulty in obtaining a numerical solution by conventional techniques. Hence, semi-empirical numerical techniques assume lot of importance with regard to these jet flows.

Formulations employing Reichardt's hypothesis considerably simplify the prediction of turbulent, incompressible, three-dimensional jet flows, and have been used by several earlier researchers /1-3/. One of these methods /3/ is being referred here as

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"*point source method*". The basic principle behind these techniques is to express the momentum flux at any point in the flow field due to an elemental source. The jet exit geometry may then be considered as an assemblage of many such elemental sources. Since the expression for momentum flux at a point is linear, the momentum fluxes from all elemental sources may be summed to yield the total momentum flux at that point.

In the present study, jets of rectangular, elliptic, and triangular exit geometries have been simulated using the point source method. The results obtained for the circular jet have been compared with the results obtained using finite difference technique. Further, the results have also been compared with experimental data /4,5/.

This paper is organized as follows: Section 2 briefly outlines the point source concept and the simulation procedure. The details of the finite difference method are also presented. Section 3 presents the simulation results and comparisons. Section 4 concludes the paper.

2. Simulation Methodology

The point source method has been described in detail by Narain /1/, Pani and Dash /2/, and Pani and Parameswaran /3/, and hence only its implementation is discussed in this section.

Consider the flow from a rectangular orifice shown in Fig.1. Let the length and height of the orifice be $2L_0$ and $2B_0$, respectively. Consider an elemental source of area $dL\ x\ dB$ around a point $(0,L,B)$ from the origin. The momentum at any point $P(x,y,z)$ in the flow field due to this elemental source is given by Eq.(1).

$$\frac{U^2}{U_0^2} = \int_{-L_0}^{L_0}\int_{-B_0}^{B_0} \frac{1}{2\pi c^2}\exp\left[-\frac{1}{2}\left\{\left(\frac{y-L}{c}\right)^2 + \left(\frac{z-B}{c}\right)^2\right\}\right] dL\,dB \qquad (1)$$

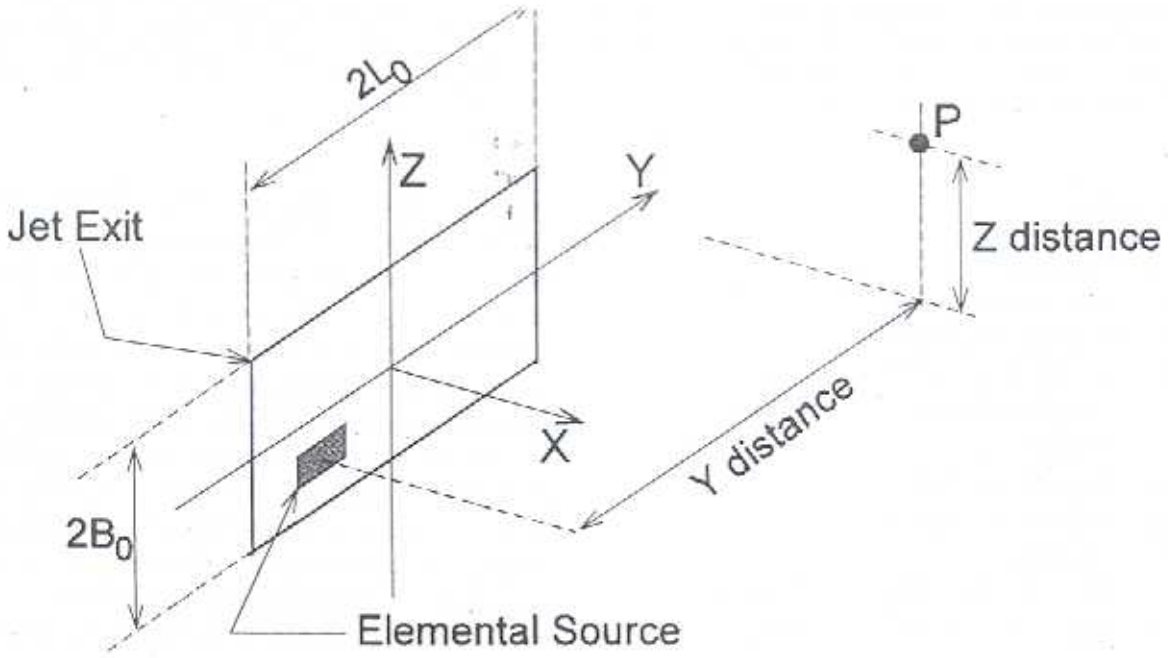


Fig. 1: The coordinate system used in the point source method

The terms $(y - L)$, and $(z - B)$ in Eq.(1), represent the distances along the y and z-axis, respectively, between the point P and the elemental source (see Fig.1). The summation is carried out for all the elemental sources, and the velocity at the point P is calculated. In the case of rectangular jets, the method is straight forward, and poses no difficulty. In the case of elliptic or triangular jets, a simplifying algorithm was used to adapt the technique. A rectangular grid was superimposed on the jet geometry, and if the centre of an elemental source fell within the jet geometry, it was considered as a contributing source. The grid was sufficiently resolved till the sum of the areas of all contributing source elements fell within 98 - 100 % of the area of the jet exit area. In order to effect a proper comparison of the various non-circular jet geometries, the areas of the jet orifices were kept the same. The diameter of a circle with the same area as the non-circular jet is being referred as the *equivalent diameter*. All linear distances have been non-dimensionalized with equivalent diameter, a term introduced by Hussain and Husain /6/. The centre of gravity was chosen to be the origin for all the jet geometries.

3. Results and Discussions

The gross features of the jets have been studied by obtaining the following:

- Decay of velocity along the jet centreline, and
- Growth of half-velocity width with axial distance, which is a measure of jet spread and mixing.

All the experimental results used for comparison in the present study have been taken from the results obtained by Srinivasan /4/. These experimental studies had been carried out at the jet test facility at high speed aerodynamics laboratory, IIT Kanpur.

3.1 Rectangular Jets

Figure 2 shows the decay of the centreline velocity for rectangular jets of aspect ratios 1, 2, 4, and 6, along with experimental data. The agreement between the predictions and the experimental data is satisfactory, and the feature of decrease in core length with increase in the aspect ratio is also correctly predicted by the point source method. For instance, while the core length of AR 1:1 jet is approximately $4D$ (Fig. 2(a)), it is approximately $2.5D$ for AR 4:1 jet (Fig. 2(c)).

Figure 3 depicts the spread of the above rectangular jets. The agreement is close in the case of the square jet (AR 1:1) (Fig. 3(a)), but simulation results deviate from experimental results as aspect ratio increases (Figs. 3(b-d)). This is because the point source method is unable to capture the phenomenon of axis-switching, which is usually observed in such jets. This phenomenon is seen from experimental results shown in Fig. 3(b-d), where the spread along the minor axis exceeds the spread along the major axis, resulting in the major and minor axes getting interchanged. However, the predictions deviate from experimental observations only along the major-axis plane. This is due to the fact that the major axis shrinks in a rectangular jet initially, and then begins to expand, while the point-source method always assumes that the jet continuously expands along both the axes. Even while doing so, the point source method demonstrates one important property of free shear flows: attainment of axisymmetry far downstream (around X/D = 30). The point-source method achieves axisymmetry in the jet by making the major and the minor-axis planes spread at different rates, monotonically.

3.2 Elliptic Jets

Figures 4(a-d) show the centreline variation of velocity for elliptic jets of aspect ratios 1, 2, 3, and, 4, respectively. The experimental results have also been shown along with the predictions, and the agreement is good. Figure 5 shows the spreading of the elliptic jets. The trends are similar to those obtained for rectangular jets. Also, the phenomenon of axis switching is applicable to elliptic jets as in the case of rectangular jets, and which may be observed from experimental data presented in Fig.5(b-d).

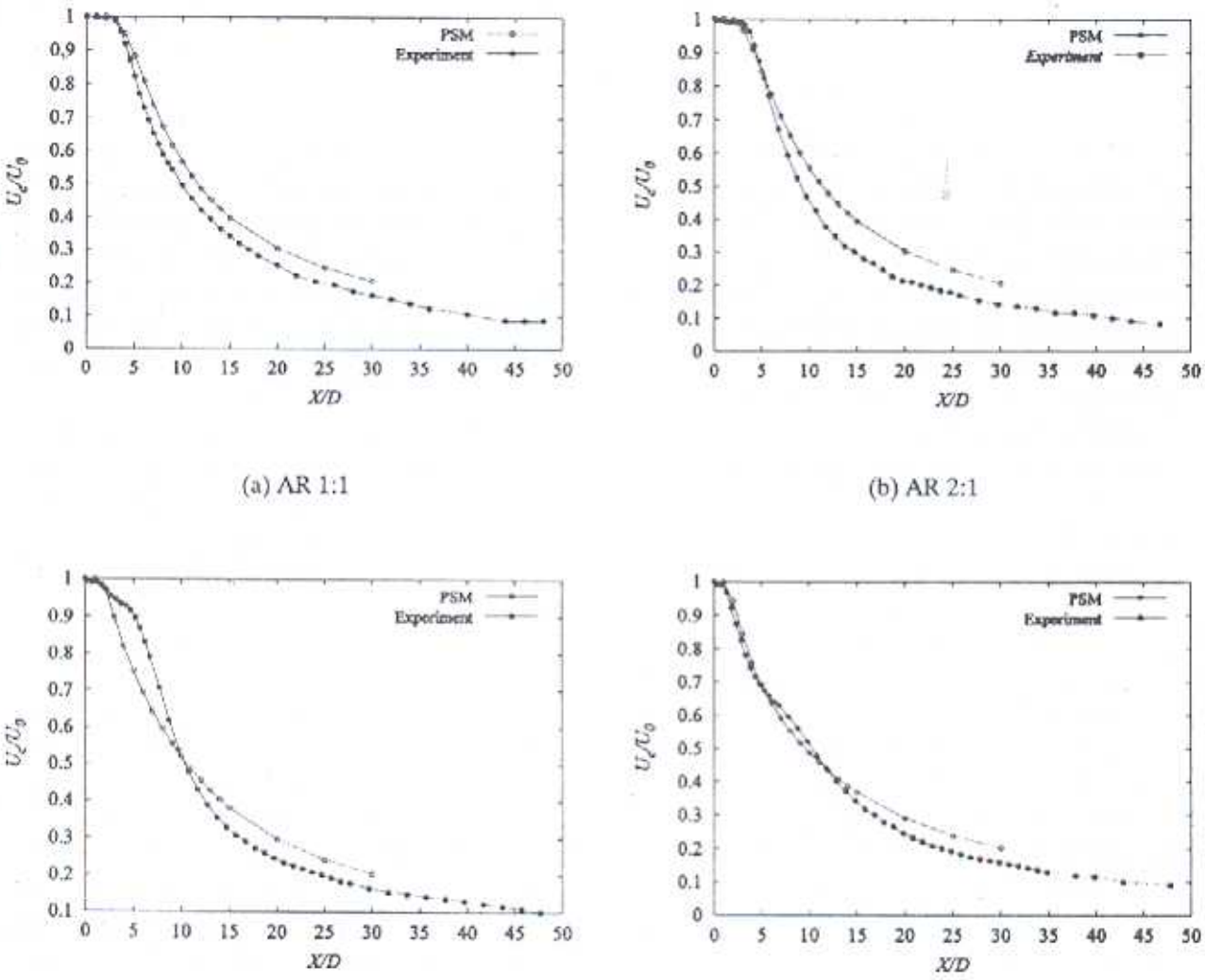


Fig. 2: Comparison of point source method predictions (PSM) with experiments in respect to centreline velocity decay of rectangular jets of various aspect ratios

3.3 Comparisons with Finite Difference Method

Plane and axisymmetric jets were simulated using finite difference method using a semi-implicit differencing scheme outlined by Hornbeck /7/. In Figure 6, plane jet and circular jet computed by finite difference method have been compared with the results obtained by point-source method, and experiments. For comparison purposes, the following three cases have been selected: (1) AR 8:1 rectangular jet computed by point source method, (2) plane jet predicted by finite difference method, and (3) AR 6:1 jet experimentally studied by Srinivasan /4/.

Figure 6(a) shows the variation of centreline velocity with non-dimensional axial distance for plane/rectangular jets. While centreline velocities obtained by the three methods seem to closely match in the near field, i.e., up to 20 jet widths, they start

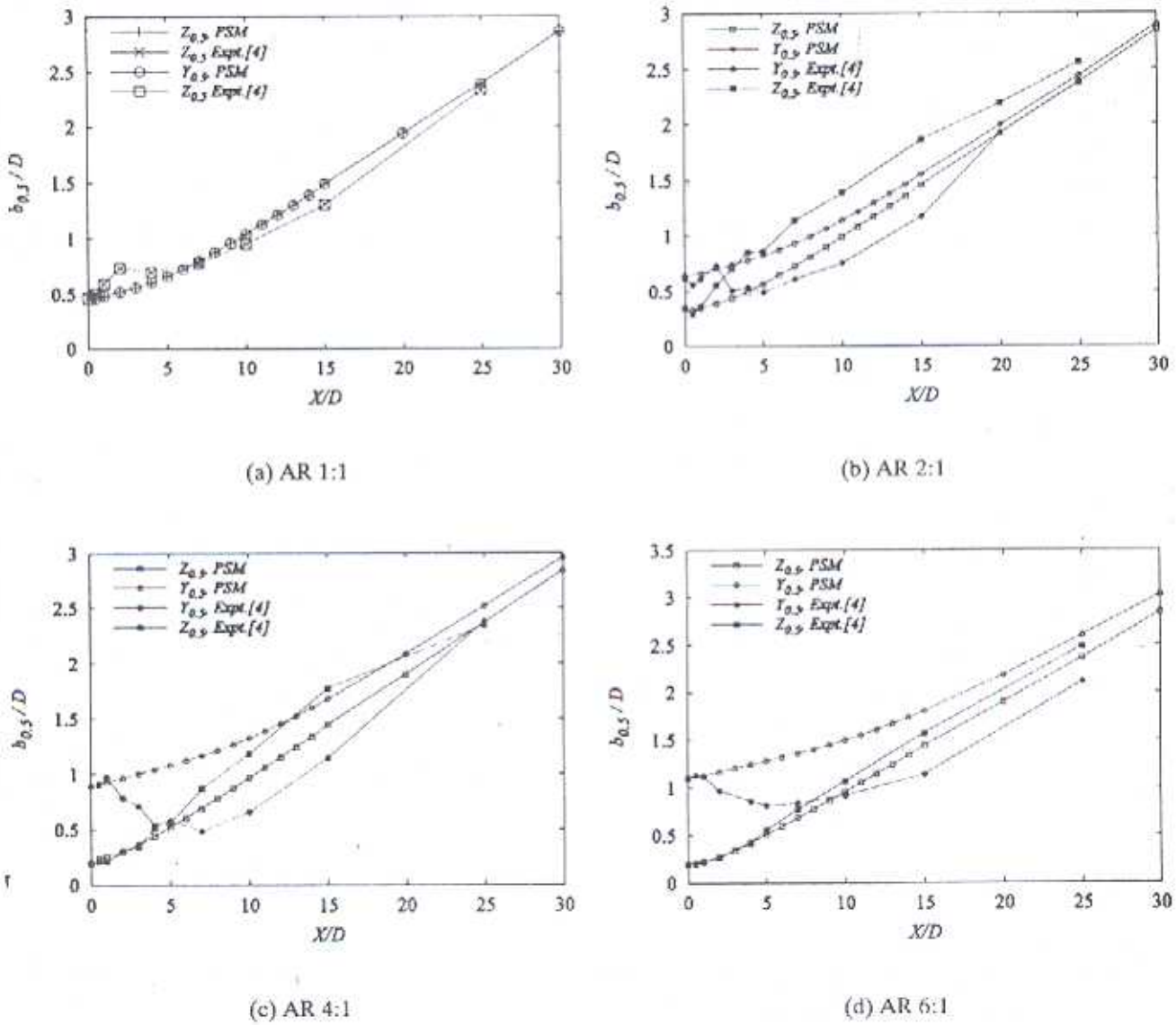
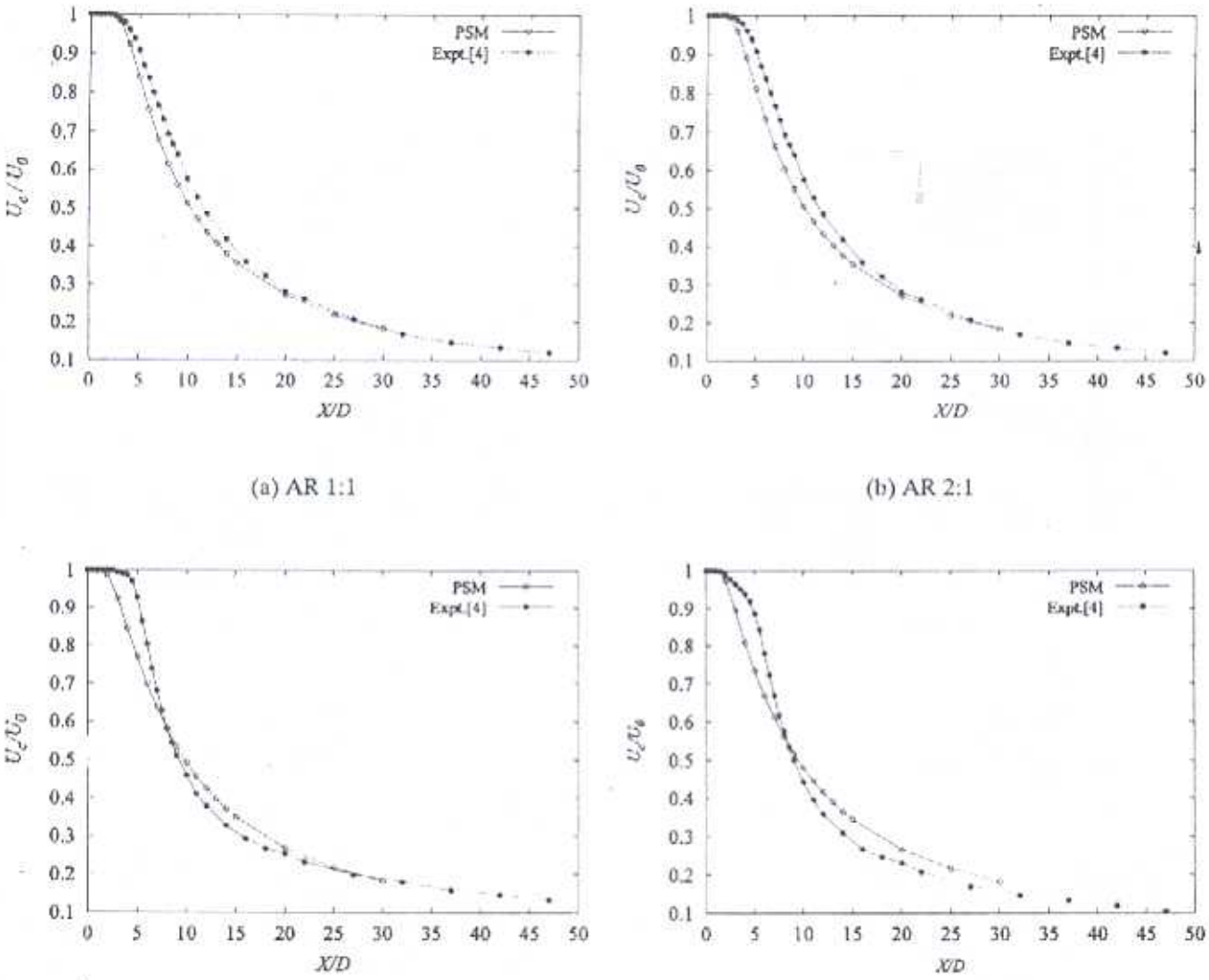


Fig. 3: Comparison of PSM results with experiments in respect to spread of rectangular jets along major and minor axes

deviating in the far field. Figure 6(b) shows the variation of half velocity width with axial distance. The results match very closely with each other, both in the near and the far fields. Figure 6(c) shows the centreline velocity variation with axial distance for circular jets. The results obtained from the two methods match fairly well with the experimental results. Despite the jet spreads not following the experimental data, it may be observed that point source technique is superior to finite difference method. The superiority of the point source method may be explained as follows: The finite difference technique is based on the assumption of axisymmetry, which reduces the problem to two dimensions, whereas the point source technique treats a simple round geometry as a three dimensional source, which is closer to reality, especially since the dynamics of the jet shear layer is necessarily a three dimensional phenomenon.



(a) AR 1:1

(b) AR 2:1

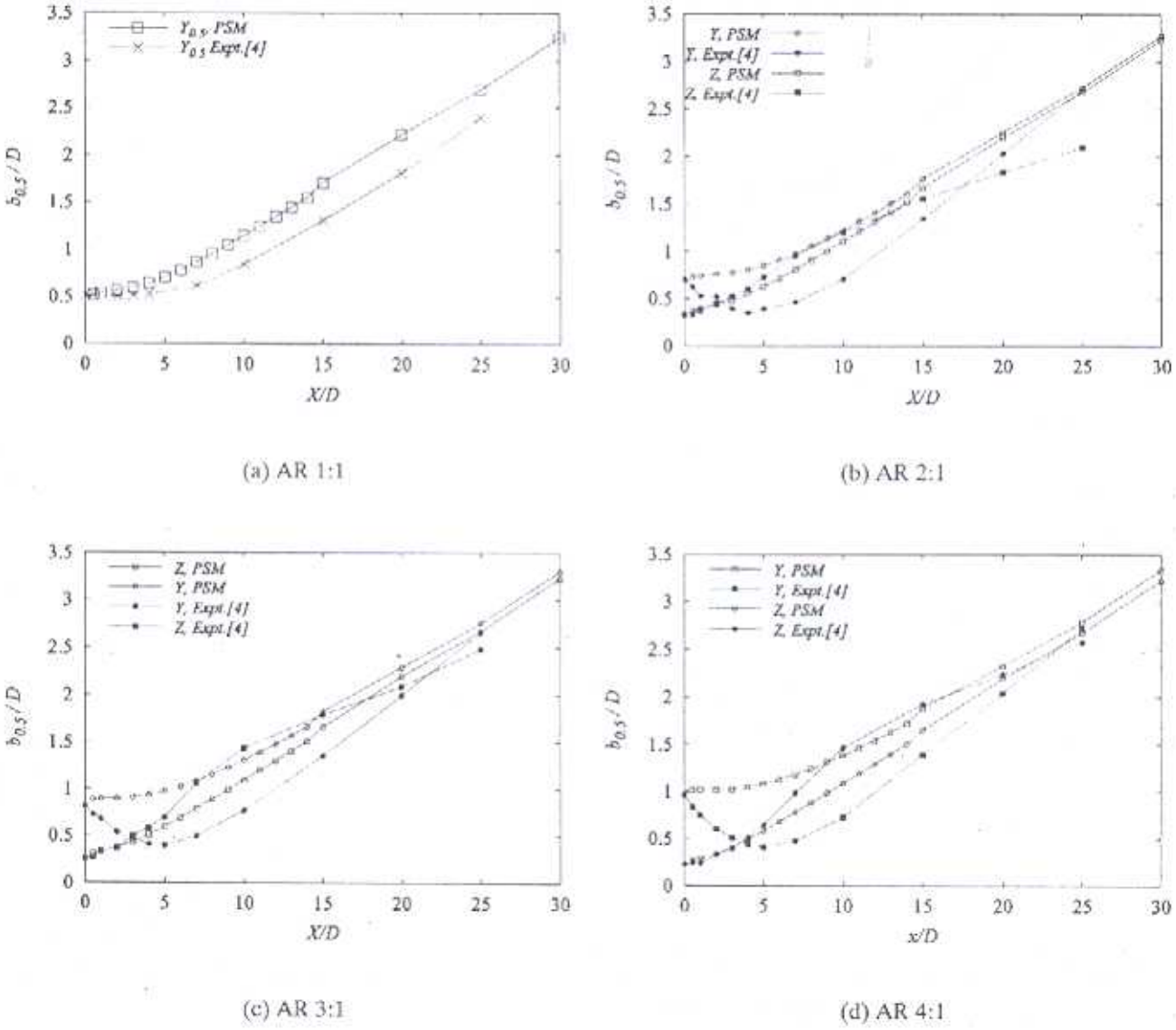
(c) AR 3:1

(d) AR 4:1

Fig. 5: Comparison of point source method predictions (PSM) with experiments in respect to decay of centreline velocity of elliptic jets of various aspect ratios

3.4 Triangular Jet

Figure 7(a) shows the centreline velocity decay for an equilateral triangular jet. The agreement between predictions using the point-source method and the experiments is significantly better than the previous cases. Figure 7(b) shows the variation of half velocity widths along OA (+ve Z axis (see Fig. 8)) and OB (-ve Z axis), with axial distance. The results obtained by point source method have been compared with experimental results of Srinivasan and Rathakrishnan /5/. Finite difference computations were not performed for triangular jets and hence not compared. In Figure 7(b), it may be observed that the half widths closely match with experimental data in the -ve Z direction compared to +ve Z direction. This is due to the fact that +ve Z



(a) AR 1:1

(b) AR 2:1

(c) AR 3:1

(d) AR 4:1

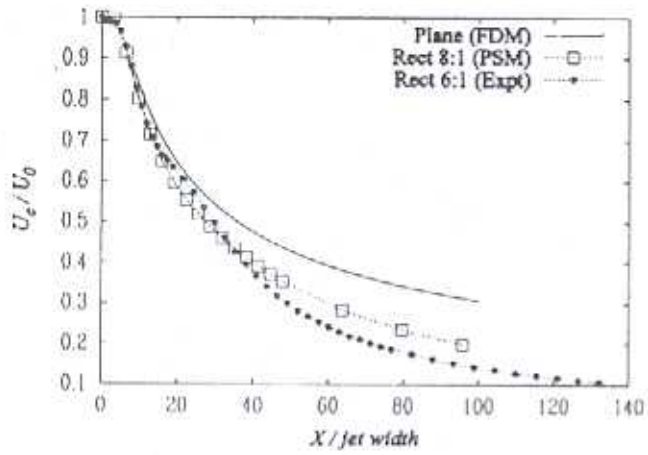
Fig. 5: Comparison of PSM results with experiments in respect to spread of elliptic jets of various aspect ratios along major and minor axes

direction is dominated by the action of streamwise vortices emanating from the vertex A, whereas the -ve Z direction is dominated by the role of spanwise vortices emanating from the flat edge PQ, similar to a rectangular jet. Since point source method cannot handle streamwise vortices, there is a deviation from experimental data of half velocity widths in the +ve Z direction.

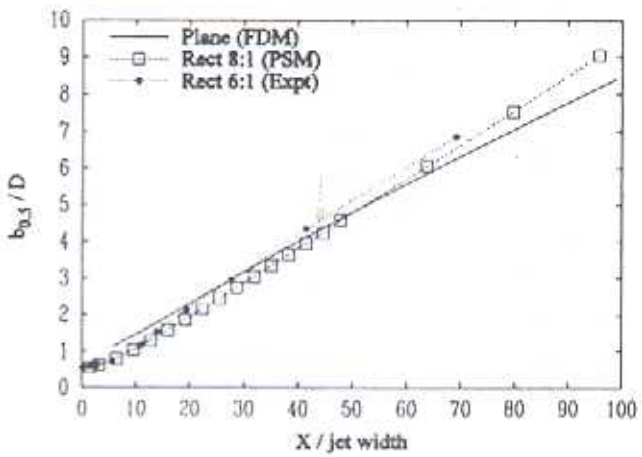
4. Conclusions

From the above results, the following conclusions may be drawn:

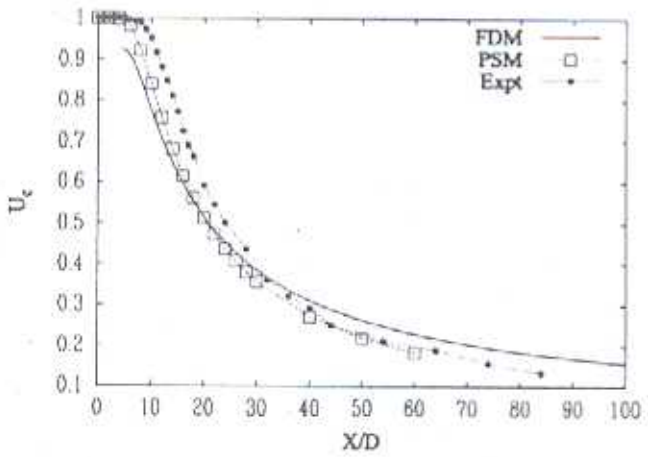
1. The variation of core length with aspect ratio for rectangular and elliptic jets can be consistently predicted using the point-source method.



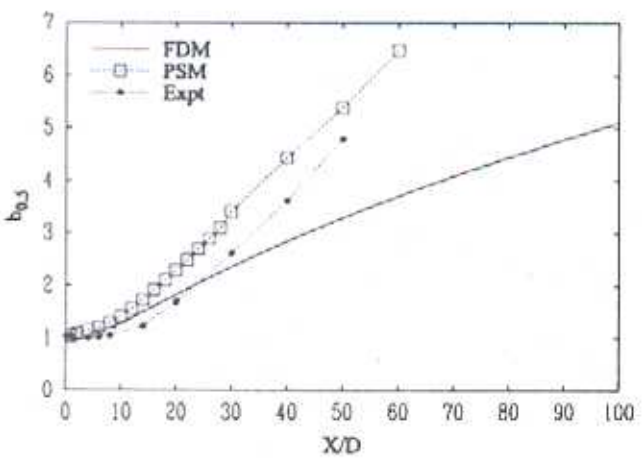
(a) Plane Jets: Centreline velocity



(b) Plane Jets: Spread



(c) Circular Jets: Centreline velocity



(d) Circular Jets: Spread

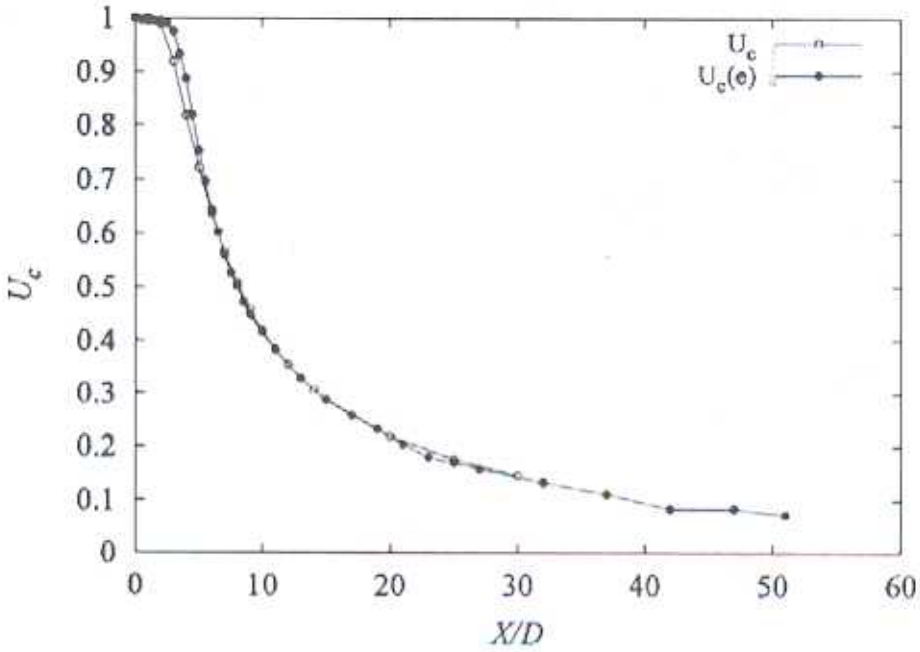
Fig. 6: Comparison between the results obtained using the point source method (PSM), finite difference method (FDM), and experimental results

2. The point-source method demonstrates the experimentally observed behaviour of attainment of axisymmetry at far downstream locations.
3. The point source method is unable to capture the commonly observed phenomenon of axis-switching in rectangular and elliptic jets.

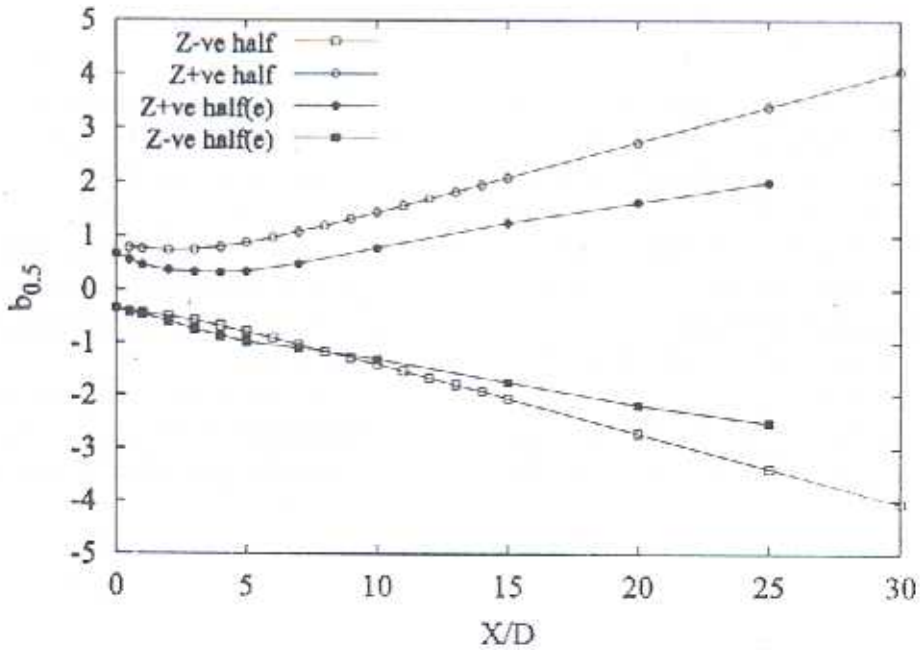
Therefore, the advantage of this technique lies in the fact that gross flow features of complex geometries can be quickly estimated for use in mixing applications in industry.

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(a) Triangular Jet: Centreline velocity



(b) Triangular Jet: Spread

Fig. 7: Comparison of PSM and experiments in respect to (a) Centreline velocity decay and (b) Spread of the triangular jet along vertex plane (Z+), and mid-base (Z-)

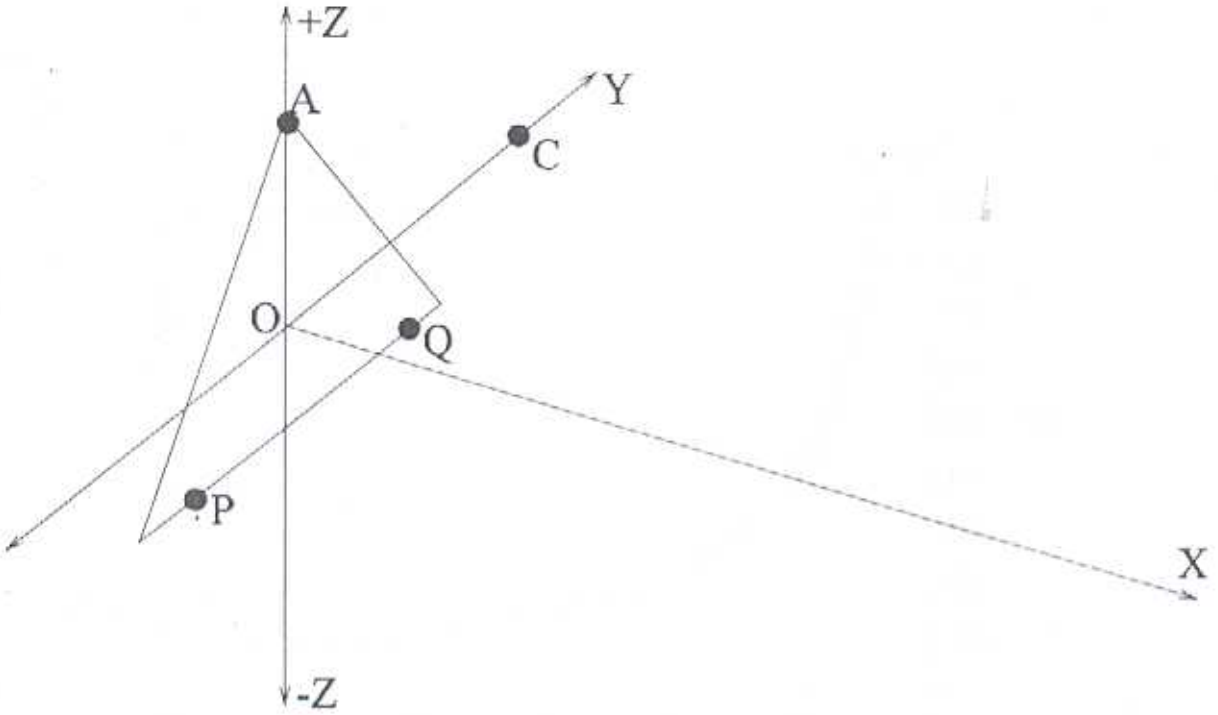


Fig. 8: Terminology used in triangular jet

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